MASTERMIND SCHOLARS EDUCATIONAL ALLIANCE

CORE MATHEMATICS

INDICES AND LOGARITHMS

Mathematics is the language of science, and indices are one of its most powerful tools. Though written as small numbers, indices play a major role in simplifying large numbers and complex expressions.

Many students find indices difficult because they focus on memorizing rules instead of understanding their meaning. This file takes a different approach by explaining what indices represent before introducing the laws. With a clear foundation, the rules become logical and easy to apply.

Indices are used widely in mathematics, science, and everyday calculations. This file guides learners step by step with simple explanations and examples, helping them gain confidence and clarity.

Welcome to a journey where indices become simple, meaningful, and easy to use.

Law 1: Multiplying indices

When multiplying indices with the same base, add the powers.

$$a^m \times a^n = a^{m+n}$$

Law 2: Quotient Law of indices

When dividing indices with the same base, subtract the powers.

$$a^m \div a^n = a^{m-n}$$

Law 3: Brackets with indices

When a number raised to an index is raised again to another index, the indices are multiplied. Therefore,

$$(a^m)^n = a^{m \times n}$$

Law 4: Power of 0

Any non-zero value raised to the power of 0 is equal to 1.

$$a^0 = 1$$

Law 5: Negative indices

When a number is raised to a negative index, it is written as the reciprocal of the number raised to the corresponding positive index.

$$a^{-m} = \frac{1}{a^m}$$

Law 6: Fractional indices

When an expression is raised to a fractional index, the denominator of the fraction represents the root, while the numerator represents the power. Therefore, $x^{\frac{a}{b}}$ is equal to the bth root of   raised to the power a.

$$x^{\frac{a}{b}} = \left(\sqrt[b]{x^a}\right)$$

LAW 7 : INVERSE OF FRACTIONS

When a fraction is raised to a negative index, the fraction is inverted and the index is made positive.

Therefore, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

EXAMPLE 1

$$\frac{7^6 \times 7^{-3}}{7^4}$$

Solution

$$\frac{7^6 \times 7^{-3}}{7^4}$$

Applying Product law of indices ($a^m \times a^n = a^{m+n}$)

$$\frac{7^{6+(-3)}}{7^4}$$

$$\frac{7^3}{7^4}$$

Apply the quotient law of indices ($a^m \div a^n = a^{m-n}$)

$$7^{3-4}$$

$$7^{-1}$$

Negative power law ($a^{-m} = \frac{1}{a^m}$)

$$7^{-1} = \frac{1}{7}$$

EXAMPLE 2

$5^n = k$ Find 5^{n+1}

Solution

$$5^{n+1}$$

$$5^n \times 5^1$$

But $5^n = k$

$$k \times 5$$

$$5k$$

EXAMPLE 3

$2^{-n} = x$ Find 2^{n+1}

Solution

$$2^{n+1}$$

$$2^n \times 2^1$$

But $2^{-n} = x$

$$\frac{1}{2^n} = x$$

$$\frac{1}{x} = 2^n$$

$$\frac{1}{x} \times 2$$

$$\frac{2}{x}$$

EXAMPLE 4

$$\left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \left(\frac{100}{81}\right)^{\frac{1}{2}}$$

Solution

$$\left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \left(\frac{100}{81}\right)^{\frac{1}{2}}$$

$$inverse\ of\ fraction\ law\ \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$$

$$\left(\frac{81}{16}\right)^{\frac{3}{4}} \times \left(\frac{100}{81}\right)^{\frac{1}{2}}$$

$$\left(\frac{3^4}{2^4}\right)^{\frac{3}{4}} \times \left(\frac{10^2}{9^2}\right)^{\frac{1}{2}}$$

$$\frac{3^{4\times\frac{3}{4}}}{2^{4\times\frac{3}{4}}} \times \frac{10^{2\times\frac{1}{2}}}{9^{2\times\frac{1}{2}}}$$

$$\frac{3^3}{2^3} \times \frac{10}{9}$$

$$\frac{27}{8} \times \frac{10}{9}$$

$$\frac{3}{4} \times \frac{5}{1} =$$

$$\frac{15}{4}$$

EXAMPLE 5

$$\frac{81^{\frac{3}{4}} - 27^{\frac{1}{3}}}{3 \times 2^3}$$

Solution

$$\frac{3^{4\times\frac{3}{4}} - 3^{3\times\frac{1}{3}}}{3 \times 2^3}$$

$$\frac{3^3 - 3}{3 \times 2^3}$$

$$\frac{27 - 3}{3 \times 8}$$

$$\frac{24}{24}$$

1

EXAMPLE 6

$$\left(\frac{1}{16}\right)^{-\frac{1}{2}} + \left(\frac{8}{27}\right)^{\frac{2}{3}}$$

Solution

$$(16)^{\frac{1}{2}} + \left(\frac{8}{27}\right)^{\frac{2}{3}}$$

$$4^{2\times\frac{1}{2}} + \left(\frac{2^3}{3^3}\right)^{\frac{2}{3}}$$

$$4^{2\times\frac{1}{2}} + \left(\frac{2^3}{3^3}\right)^{\frac{2}{3}}$$

$$4^{2\times\frac{1}{2}} + \frac{2^{3\times\frac{2}{3}}}{3^{3\times\frac{2}{3}}}$$

$$4+\frac{2^2}{3^2}$$

$$4+\frac{4}{9}$$

$$\frac{40}{9}$$

EXAMPLE 7

$$\frac{2^{n+4}-6\times 2^{n-1}}{2^{2(n+1)}}$$

Solution

$$\frac{(2^n\times 2^4)-6\times\left(\frac{2^n}{2}\right)}{2^{2n+2}}$$

$$\frac{(2^n\times 2^4)-3\times 2^n}{2^{2n}\times 2^2}$$

$$\frac{(2^n\times 16)-(3\times 2^n)}{(2^n)^2\times 4}$$

$$\frac{16(2^n)-3(2^n)}{4(2^n)^2}$$

$$\frac{13(2^n)}{4(2^n)^2}$$

$$\frac{13}{4(2^n)}$$

EXAMPLE 8

$$\frac{2^{n+4}-6\times 2^{n-1}}{2^{(n+1)}}$$

Solution

$$\frac{(2^n\times 2^4)-6\times\left(\frac{2^n}{2}\right)}{2^{n+1}}$$

$$\frac{(2^n\times 2^4)-3\times 2^n}{2^n\times 2}$$

$$\frac{(2^n \times 16) - (3 \times 2^n)}{2^n \times 2}$$

$$\frac{16(2^n) - 3(2^n)}{2(2^n)}$$

$$\frac{13(2^n)}{2(2^n)}$$

$$\frac{13}{2}$$

EXAMPLE 9

$$\sqrt{\frac{8^2 \times 4^{n-1}}{2^{2n} \times 16}}$$

Solution

$$\sqrt{\frac{2^{3(2)} \times 2^{2(n-1)}}{2^{2n} \times 2^4}}$$

$$\sqrt{\frac{2^6 \times 2^{2n-2}}{2^{2n} \times 2^4}}$$

$$\sqrt{\frac{2^{6+2n-2}}{2^{2n+4}}}$$

$$\sqrt{\frac{2^{2n+4}}{2^{2n+4}}}$$

$$\sqrt{1}$$

$$1$$

EXAMPLE 10

$$\frac{3^{n+1} \times 27^{n+1}}{81^n}$$

Solution

$$\frac{3^{n+1} \times 27^{n+1}}{81^n}$$

$$\frac{3^{n+1} \times 3^{3(n+1)}}{3^{4n}}$$

$$\frac{3^{n+1} \times 3^{3n+3}}{3^{4n}}$$

$$\frac{3^{(n+1)+(3n+3)}}{3^{4n}}$$

$$\frac{3^{4n+4}}{3^{4n}}$$

$$3^{(4n+4)-4n}$$

$$3^4$$

$$81$$

EXAMPLE 11

$$\left[\left(\frac{16}{9}\right)^{-\frac{3}{4}} \times 16^{-\frac{3}{4}}\right]^{\frac{1}{3}}$$

Solution

$$\left[\left(\frac{9}{16}\right)^{\frac{3}{4}} \times \left(\frac{1}{16}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}$$

$$\left[\left(\frac{3^2}{2^4}\right)^{\frac{3}{4}} \times (2^{-4})^{\frac{3}{4}}\right]^{\frac{1}{3}}$$

$$\left[\left(\frac{3^{2\times\frac{3}{4}}}{2^{4\times\frac{3}{4}}}\right) \times (2^{-4})^{\frac{3}{4}}\right]^{\frac{1}{3}}$$

$$\left[\left(\frac{3^{\frac{3}{2}}}{2^3}\right) \times 2^{-3}\right]^{\frac{1}{3}}$$

$$\left[\left(\frac{3^{\frac{3}{2}}}{2^3}\right) \times \frac{1}{2^3}\right]^{\frac{1}{3}}$$

$$\left[\left(\frac{3^{\frac{3}{2}}}{2^{3+3}}\right)\right]^{\frac{1}{3}}$$

$$\left[\left(\frac{3^{\frac{3}{2}}}{2^6}\right)\right]^{\frac{1}{3}}$$

$$\left(\frac{3^{\frac{3}{2}\times\frac{1}{3}}}{2^{6\times\frac{1}{3}}}\right)$$

$$\left(\frac{3^{\frac{1}{2}}}{2^2}\right)$$

$$\frac{\sqrt{3}}{4}$$

EXAMPLE 12

$$5^{3x+5} = \frac{1}{25}$$

Solution

$$5^{3x+5} = \frac{1}{25}$$

Represent all numbers to have the same base

$$5^{3x+5} = \frac{1}{5^2}$$

$$5^{3x+5} = 5^{-2}$$

Since the bases are the same equate the powers

$$3x + 5 = -2$$

$$3x = -2 - 5$$

$$3x = -7$$

$$x = -\frac{7}{3}$$

EXAMPLE 13

$$\frac{2^{3x+5}}{8} = 2^{2x}$$

Solution

$$\frac{2^{3x+5}}{2^3} = 2^{2x}$$

$$\frac{2^{3x+5}}{2^3} = 2^{2x}$$

Simplify the LHS with the laws if indices

$$2^{(3x+5)-3} = 2^{2x}$$

Since bases are the same, equate the powers

$$(3x + 5) - 3 = 2x$$

$$3x + 5 - 3 = 2x$$

$$3x + 2 = 2x$$

$$3x - 2x = -2$$

$$x = -2$$

EXAMPLE 14

$$\frac{64^{x+1}}{4^x} = 16^{2x-1}$$

Solution

$$\frac{64^{x+1}}{4^x} = 16^{2x-1}$$

$$\frac{4^{3(x+1)}}{4^x} = 4^{2(2x-1)}$$

$$\frac{4^{3x+3}}{4^x} = 4^{4x-2}$$

Simplify the LHS using the laws of indices

$$4^{3x+3-x} = 4^{4x-2}$$

Since bases are the same equate the powers

$$3x + 3 - x = 4x - 2$$

$$3x - x - 4x = -2 - 3$$

$$-2x = -5$$

$$x = \frac{-5}{-2}$$

$$x = \frac{5}{2}$$

EXAMPLE 15

$(3n-1)^{\frac{1}{3}} = 2$ Find the value of n

Solution

$$(3n-1)^{\frac{1}{3}} = 2$$

$$(3n-1)^{\frac{1}{3}\times 3} = 2^3$$

$$3n-1 = 8$$

$$3n = 9$$

$$n = \frac{9}{3}$$

$$n = 3$$

EXAMPLE 16

$$9^{y+1} = \left(\frac{1}{27}\right)^{y-2}$$

Solution

$$9^{y+1} = \left(\frac{1}{27}\right)^{y-2}$$

$$3^{2(y+1)} = \left(\frac{1}{3^3}\right)^{y-2}$$

$$3^{2(y+1)} = 3^{-3^{(y-2)}}$$

Since the bases are the same equate the powers

$$3^{2(y+1)} = 3^{-3^{(y-2)}}$$

$$2(y+1) = -3(y-2)$$

$$2y+2 = -3y+6$$

$$2y+3y = 6-2$$

$$5y = 4$$

$$y = \frac{4}{5}$$

EXAMPLE 17

$$5^{3(x-3)} = \sqrt{5^{4x-7}}$$

Solution

$$5^{3(x-3)} = \sqrt{5^{4x-7}}$$

Square both side to remove the square root

$$\left(5^{3(x-3)}\right)^2 = \left(\sqrt{5^{4x-7}}\right)^2$$

$$(5^{3x-9})^2 = 5^{4x-7}$$

$$5^{2(3x-9)} = 5^{4x-7}$$

Since the bases are the same , equate the powers

$$2(3x-9) = 4x-7$$

$$6x-18 = 4x-7$$

$$6x-4x = -7+18$$

$$2x = 11$$

$$x = \frac{11}{2}$$

EXAMPLE 18

$$\frac{3^{2x-1}}{3^{3x-4} \times 3^{6-7x}} = 27^x$$

Solution

$$\frac{3^{2x-1}}{3^{3x-4} \times 3^{6-7x}} = 3^{3x}$$

$$\frac{3^{2x-1}}{3^{3x-4+6-7x}} = 3^{3x}$$

$$\frac{3^{2x-1}}{3^{-4x+2}} = 3^{3x}$$

$$3^{2x-1-(-4x+2)} = 3^{3x}$$

$$3^{2x-1+4x-2} = 3^{3x}$$

$$3^{6x-3} = 3^{3x}$$

Since bases are the same equate the powers

$$6x-3 = 3x$$

$$6x-3x = 3$$

$$3x = 3$$

$$x = \frac{3}{3}$$

$$x = 1$$

EXAMPLE 19

$$16 \times 2^{(x+1)} = 4^x \times 8^{(1-x)}$$

Solution

$$16 \times 2^{(x+1)} = 4^x \times 8^{(1-x)}$$

$$2^4 \times 2^{(x+1)} = 2^{2x} \times 2^{3(1-x)}$$

$$2^4 \times 2^{(x+1)} = 2^{2x} \times 2^{3(1-x)}$$

Using the product law of indices add the powers for both LHS and RHS

$$2^{4+(x+1)} = 2^{2x+3(1-x)}$$

Since the bases are the same , equate the powers

$$4 + (x + 1) = 2x + 3(1 - x)$$

$$4 + x + 1 = 2x + 3 - 3x$$

$$x - 2x + 3x = 3 - 4 - 1$$

$$2x = -2$$

$$x = -\frac{2}{2}$$

$$x = -1$$

EXAMPLE 20

$3^{(x+y)} = 243$ and $3x + y = 9$, find the value of x and y

Solution

$$3^{(x+y)} = 243$$

$$3^{(x+y)} = 3^5$$

$x + y = 5$ ---------------- eqn 1

$3x + y = 9$ ------------------- eqn 2

Solving eqn 1 and 2 simultaneously using elimination

x + y = 5

$3x + y = 9$

$-2x = -4$

$x = 2$

Subtitute x = 2 into any of the two equations

2 + y = 5

y = 5 - 2

y = 3

EXAMPLE 21

$2^x \times 2^y = 16$ and $3^{2x} \div \left(\frac{1}{3}\right)^{y-1} = 81$, find the value of x and y

Solution

$$2^x \times 2^y = 16$$

$$2^x \times 2^y = 2^4$$

$$2^{x+y} = 2^4$$

Since the bases are the same equate the powers

$x + y = 4$ -------------- eqn 1

$$3^{2x} \div \left(\frac{1}{3}\right)^{y-1} = 81$$

$$3^{2x} \div \left(\frac{1}{3}\right)^{y-1} = 3^4$$

$$3^{2x} \div 3^{-(y-1)} = 3^4$$

Subtract the powers since the bases are the same and dividing on the LHS

$$3^{2x-(-y+1)} = 3^4$$

$$3^{2x+y-1)} = 3^4$$

$$3^{2x+y-1} = 3^4$$

Since the bases are the same equate the powers

$$2x + y - 1 = 4$$

$$2x + y = 4 + 1$$

$2x + y = 5$ ------------------ eqn 2

Solving eqn 1 and eqn 2 simultaneously using elimination method

$x + y = 4$ -------------- eqn 1

$$2x + y = 5 \text{ ------------------ eqn 2}$$

$$-x = -1$$

$$x = 1$$

Substituting $x = 1$ into equation 1

$$x + y = 4$$

$$1 + y = 4$$

$$y = 4 - 1$$

$$y = 3$$

Solving eqn 1 and eqn 2 simultaneously using substitution method

$$x + y = 4 \text{ -------------- eqn 1}$$

$$2x + y = 5 \text{ ------------------ eqn 2}$$

Making x the subject of equation 1

$$x + y = 4$$

$$x = 4 - y$$

Substituting the result into equation

$$2x + y = 5$$

But $x = 4 - y$

$$2(4 - y) + y = 5$$

$$8 - 2y + y = 5$$

$$8 - y = 5$$

$$-y = 5 - 8$$

$$-y = -3$$

$$y = 3$$

Substitute y into the But $x = 4 - y$

$$x = 4 - 3$$

$$x = 1$$

EXAMPLE 22

5x – 4y = 6

$$3^{3(y-x)} = \frac{1}{3^3}$$

Find the value of x and y

5x – 4y = 6 ------------- eqn 1

$$3^{3(y-x)} = \frac{1}{3^3}$$

$$3^{3y-3x} = 3^{-3}$$

Since the bases are the same equate the powers

$$3y - 3x = -3$$

$$y - x = -1$$

$$-x + y = -1$$

$x - y = 1$ ---------- equation 2

Solving simultaneously using substitution

5x – 4y = 6 ------------- eqn 1

$x - y = 1$ ---------- equation 2

Make x the subject of equation 2

$$x - y = 1$$

$$x = 1 + y$$

Substitute the value of x into equation 1

$$5x - 4y = 6$$

$$5(1 + y) - 4y = 6$$

$$5 + 5y - 4y = 6$$

$$5 + y = 6$$

$$y = 6 - 5$$

$$y = 1$$

Substitute the value of y into any of the equation

$$x - 1 = 1$$

$$x = 1 + 1$$

$$x = 2$$

INDICES QUADRATIC EQUATIONS

EXAMPLE 23

$$8^{\left(x-\frac{2}{3}\right)} = 2^{x^2}$$

Solution

$$8^{\left(x-\frac{2}{3}\right)} = 2^{x^2}$$

$$2^{3\left(x-\frac{2}{3}\right)} = 2^{x^2}$$

$$2^{3\left(x-\frac{2}{3}\right)} = 2^{x^2}$$

Since the bases are the same equate the powers

$$3\left(x-\frac{2}{3}\right) = x^2$$

$$3x - 2 = x^2$$

Representing the equation into standard quadratic form

$$x^2 - 3x + 2 = 0$$

$$x^2 - x - 2x + 2 = 0$$

$$x(x-1) - 2(x-1) = 0$$

$$(x-2)(x-1) = 0$$

$$x - 2 = 0 \quad \text{and } x - 1 = 0$$

$$x = 2 \qquad\qquad x = 1$$

EXAMPLE 24

$$2^{8(x+1)} = 2^{3(1-x^2)}$$

Solution

$$2^{8(x+1)} = 2^{3(1-x^2)}$$

Since the bases are the same we equate the powers

$$8(x+1) = 3(1-x^2)$$

$$8x + 8 = 3 - 3x^2$$

$$-3x^2 - 8x - 8 + 3$$

$$-3x^2 - 8x - 5$$

Multiply 15

Add -8

$$-3 \; and \; -5$$

$$-3x^2 - 3x - 5x - 5$$

$$-3x(x+1) - 5(x+1)$$

$$(-3x-5)(x+1)$$

$$-3x - 5 = 0$$

$$-3x = 5$$

$$x = -\frac{5}{3}$$

$$x + 1 = 0$$

$$x = -1$$

EXAMPLE 25

$3^{2x} - 4(3^x) + 3 = 0$ Find the value of x

Solution

$$3^{2x} - 4(3^x) + 3 = 0$$

$$(3^x)^2 - 4(3^x) + 3 = 0$$

Let $3^x = y$

$$y^2 - 4y + 3 = 0$$

$$y^2 - 3y - y + 3 = 0$$

$$y(y - 3) - (y - 3) = 0$$

$$(y - 1)(y - 3) = 0$$

y = 1 and y =3

If y = 1

Let $3^x = y$

$3^x = 1$

$3^x = 3^0$

x = 0

If y = 3

Let $3^x = y$

$3^x = 3$

$3^x = 3^1$

x = 1

EXAMPLE 26

$2^{(2x+1)} - 9(2^x) + 4 = 0$ Find the value of x

Solution

$$2^{(2x+1)} - 9(2^x) + 4 = 0$$

$$(2^x)^2 \times 2 - 9(2^x) + 4 = 0$$

$$2(2^x)^2 - 9(2^x) + 4 = 0$$

Let $2^x = y$

$$2(y)^2 - 9(y) + 4 = 0$$

$$2y^2 - 9y + 4 = 0$$

$$2y^2 - 8y - y + 4 = 0$$

$$2y(y-4) - (y-4) = 0$$

$$(2y-1)(y-4) = 0$$

2y = 1 and y = 4

y = $\frac{1}{2}$ and y = 4

If y = $\frac{1}{2}$

Let $2^x = \frac{1}{2}$

$2^x = 2^{-1}$

x = -1

If y = 4

Let $2^x = 4$

$2^x = 2^2$

x = 2

EXAMPLE 27

$4(3^{2x+1}) + 17(3^x) - 7 = 0$ Find the value of x

Solution

$$4(3^{2x+1}) + 17(3^x) - 7 = 0$$

$$4[(3^x)^2 \times 3] + 17(3^x) - 7 = 0$$

Let $3^x = y$

$$4[(y)^2 \times 3] + 17(y) - 7 = 0$$

$$4[3(y)^2] + 17y - 7 = 0$$

$$12y^2 + 17y - 7 = 0$$

$$12y^2 - 4y + 21y - 7 = 0$$

$$4y(3y - 1) + 7(3y - 1) = 0$$

$$(4y + 7)(3y - 1) = 0$$

$4y + 7 = 0$

$4y = -7$

$y = -\frac{7}{4}$

$3y – 1 = 0$

$3y = 1$

$y = \frac{1}{3}$

$y = -\frac{7}{4}$ and $y = \frac{1}{3}$

If $y = -\frac{7}{4}$

Let $3^x = -\frac{7}{4}$

If $y = \frac{1}{3}$

Let $3^x = \frac{1}{3}$

$3^x = 3^{-1}$

$x = -1$

LAWS OF LOGARITHMS

1. $\log_a(xy) = \log_a x + \log_a y$

2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

3. $\log_a x^n = n \log_a x$

4. $\log_a a = 1$ and $\log_{10} 1 = 0$

5. $\log_a b = \frac{1}{\log_b a}$ or $\log_a b = \frac{\log_x b}{\log_x a}$

NOTE : $\log_a -x$ *is undefined*, $\log_{-a} x$ *is undefined*

For a log function to be correctly defined then $\log_a x$ should be such that a $> 1, x > 0$

CHANGING INDICES TO LOG AND VICE VERSA

EXAMPLE 1

Indical form

8 = 2^3

Log form

$\log_2 8 = 3$

EXAMPLE 2

Indical form

64 = 4^3

Log form

$\log_4 64 = 3$

EXAMPLE 3

Indical form

$\frac{1}{125} = 5^{-3}$

Log form

$\log_5\left(\frac{1}{125}\right) = -3$

EXAMPLE 4

Write
$\log_5\left(\frac{1}{25}\right) = -2$ in indicial form

Solution

$$\frac{1}{25} = 5^{-2}$$

SIMPLIFYING LOGARITHMIC USING LAWS OF LOG

EXAMPLE 1

Given that $\log_{10}7 = 0.8451$ and $\log_{10}3 = 0.4771$ evaluate $\log_{10}21$

Solution

$$\log_{10}21$$

Applying the law of multiplication of log

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_{10}(7 \times 3)$$

$$\log_{10} 7 + \log_{10} 3$$

$$0.8451 + 0.4771$$

$$1.3222$$

EXAMPLE 2

Given that $\log_{10}7 = 0.8451$ and $\log_{10}3 = 0.4771$ evaluate $\log_{10}\left(\frac{9}{7}\right)$

Solution

$$\log_{10}\left(\frac{9}{7}\right)$$

Applying the division law of log

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_{10}\left(\frac{9}{7}\right)$$

$$\log_{10} 9 - \log_{10} 7$$

$$\log_{10} 3^2 - \log_{10} 7$$

Applying the power law of log $\log_a x^n = n \log_a x$

$$2\log_{10} 3 - \log_{10} 7$$

$$2(0.4771\ ) - \ 0.8451$$

0.9542 – 0.8451

0.1091

EXAMPLE 3

Given that $\log_{10}7 = 0.8451$ and $\log_{10}3 = 0.4771$ evaluate $\log_{10}900$

Solution

$$\log_{10}900$$

$$\log_{10}(9\ \times 100)$$

Applying the law of multiplication of log

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_{10} 9 + \log_{10} 100$$

$$\log_{10} 3^2 + \log_{10} 10^2$$

$$2\log_{10} 3 + 2\log_{10} 10$$

$$2(0.4771\ ) + 2(1)$$

0.9542 + 2

2.9542

EXAMPLE 4

Simplify

$$\log_{10}25 + \log_{10}32 - \log_{10}8$$

Solution

$$\log_{10}25 + \log_{10}32 - \log_{10}8$$

$$\log_{10}\left(\frac{25 \times 32}{8}\right)$$

$$\log_{10}\left(\frac{25 \times 32}{8}\right)$$

$$\log_{10}(100)$$

$$\log_{10} 10^2$$

$$2\log_{10} 10$$

$$2(1)$$

$$2$$

Alternative method

$$\log_{10}25 + \log_{10}32 - \log_{10}8$$

$$\log_{10}(25 \times 32) - \log_{10}8$$

$$\log_{10}(800) - \log_{10}8$$

$$\log_{10}\left(\frac{800}{8}\right)$$

$$\log_{10}(100)$$

$$\log_{10} 10^2$$

$$2\log_{10} 10$$

$$2(1)$$

$$2$$

EXAMPLE 5

Given that $\log_{10} v + 2\log_{10} m = 1$, Express V in terms of m

Solution

$$\log_{10} v + 2\log_{10} m = 1$$

$$\log_{10} v + \log_{10} m^2 = \log_{10} 10$$

$$\log_{10}(v\, m^2) = \log_{10} 10$$

Taking the antilog of both sides

$$vm^2 = 10$$

$$v = \frac{10}{m^2}$$

EXAMPLE 6

If $\log_{10} y = 3\log_{10} 2x$, what is the value of x ?

Solution

$$\log_{10} y = 3\log_{10} 2x$$

$$\log_{10} y = \log_{10}(2x)^3$$

Taking the antilog of both sides

$$y = (2x)^3$$

$$y = 8x^3$$

$$x^3 = \frac{y}{8}$$

$$x = \sqrt[3]{\frac{y}{8}}$$

$$x = \frac{\sqrt[3]{y}}{2}$$

EXAMPLE 7

Express 2 - $5\log_{10} 2$ as logarithm of a single number

Solution

$$2 - 5\log_{10} 2$$

$$\log_{10} 100 - 5\log_{10} 2$$

$$\log_{10} 100 - \log_{10} 2^5$$

$$\log_{10}\left(\frac{100}{2^5}\right)$$

$$\log_{10}\left(\frac{100}{32}\right)$$

$$\log_{10}\left(\frac{25}{8}\right)$$

EXAMPLE 8

If $\log_{10} y = 3\log_{10} 2 + \log_{10} 3 - \log_{10} 6$, find the value of y

Solution

$$\log_{10} y = 3\log_{10} 2 + \log_{10} 3 - \log_{10} 6$$

$$\log_{10} y = \log_{10} 2^3 + \log_{10} 3 - \log_{10} 6$$

$$\log_{10} y = \log_{10}\left(\frac{2^3 \times 3}{6}\right)$$

$$\log_{10} y = \log_{10}\left(\frac{8 \times 3}{6}\right)$$

$$\log_{10} y = \log_{10}\left(\frac{24}{6}\right)$$

$$\log_{10} y = \log_{10} 4$$

Taking the antilog of both sides

y = 4

EXAMPLE 9

If $log\ a = 0.3030$, $log\ b = 0.4771$ and $log\ c = 0.8451$ then ,

Evaluate : $\frac{\log a - \log c^{\frac{1}{3}}}{\log b^3}$

Solution

$$\frac{\log a - \log c^{\frac{1}{3}}}{\log b^3}$$

$$\frac{\log a - \frac{1}{3}\log c}{3\log b}$$

$$\frac{(0.3030) - \frac{1}{3}(0.8451)}{3(0.4771)}$$

$$\frac{0.3030 - 0.2817}{1.4313}$$

$$\frac{0.0213}{1.4313}$$

0.0149

EXAMPLE 10

If $\log_{10}(2x + 1) - \log_{10}(3x - 2) = 1$ find x

Solution

$$\log_{10}(2x + 1) - \log_{10}(3x - 2) = 1$$

$$\log_{10}(2x + 1) - \log_{10}(3x - 2) = \log_{10} 10$$

$$\log_{10}\left(\frac{2x + 1}{3x - 2}\right) = \log_{10} 10$$

Taking anti log of both sides

$$\frac{2x + 1}{3x - 2} = 10$$

$$2x + 1 = 10(3x - 2)$$

$$2x + 1 = 30x - 20$$

$$1 + 20 = 30x - 2x$$

$$21 = 28x$$

$$x = \frac{21}{28}$$

$$x = \frac{3}{4}$$

EXAMPLE 11

Given that $6\log(x+4) = \log 64$ find the value of x

Solution

$$6\log(x+4) = \log 64$$

$$\log(x+4)^6 = \log 64$$

$$\log(x+4)^6 = \log 64$$

Taking the antilog of both sides

$$(x+4)^6 = 64$$

$$(x+4)^{6\times\frac{1}{6}} = 2^{6\times\frac{1}{6}}$$

$$x+4 = 2$$

$$x = 2-4$$

$$x = -2$$

EXAMPLE 12

Without using mathematical tables or calculator, simplify

$$\frac{1}{2}\log_{10}\left(\frac{25}{4}\right) - 2\log_{10}\left(\frac{4}{5}\right) + \log_{10}\left(\frac{320}{125}\right)$$

Solution

$$\frac{1}{2}\log_{10}\left(\frac{25}{4}\right) - 2\log_{10}\left(\frac{4}{5}\right) + \log_{10}\left(\frac{320}{125}\right)$$

$$\log_{10}\left(\frac{25}{4}\right)^{\frac{1}{2}} - \log_{10}\left(\frac{4}{5}\right)^{2} + \log_{10}\left(\frac{320}{125}\right)$$

$$\log_{10}\left(\frac{\left(\frac{25}{4}\right)^{\frac{1}{2}} \times \left(\frac{320}{125}\right)}{\left(\frac{4}{5}\right)^{2}}\right)$$

$$\log_{10}\left(\frac{\frac{5}{2}\times\frac{64}{25}}{\frac{16}{25}}\right)$$

$$\log_{10}\left(\frac{\frac{5}{2}\times\frac{64}{25}}{\frac{16}{25}}\right)$$

$$\log_{10}\left(\frac{\frac{32}{5}}{\frac{16}{25}}\right)$$

$$\log_{10}\left(\frac{32}{5}\div\frac{16}{25}\right)$$

$$\log_{10}\left(\frac{32}{5}\times\frac{25}{16}\right)$$

$$\log_{10}\left(\frac{32}{5}\times\frac{25}{16}\right)$$

$$\log_{10}10$$

$$1$$

EXAMPLE 13

Without using mathematical tables or calculator, simplify

$$\log_{10}\left(\frac{75}{10}\right)-2\log_{10}\left(\frac{5}{9}\right)+\log_{10}\left(\frac{100}{243}\right)$$

Solution

$$\log_{10}\left(\frac{75}{10}\right)-\log_{10}\left(\frac{5}{9}\right)^2+\log_{10}\left(\frac{100}{243}\right)$$

$$\log_{10}\left(\frac{\frac{75}{10}\times\frac{100}{243}}{\left(\frac{5}{9}\right)^2}\right)$$

$$\log_{10}\left(\frac{\frac{75}{10}\times\frac{100}{243}}{\frac{25}{81}}\right)$$

$$\log_{10}\left(\frac{\frac{750}{243}}{\frac{25}{81}}\right)$$

$$\log_{10}\left(\frac{750}{243}\div\frac{25}{81}\right)$$

$$\log_{10}\left(\frac{750}{243}\times\frac{81}{25}\right)$$

$$\log_{10}\left(\frac{30}{243}\times\frac{81}{1}\right)$$

$$\log_{10}\left(\frac{2430}{243}\right)$$

$$\log_{10}10$$

$$1$$

EXAMPLE 14

Solve the equation

$$x\log 2 + \log 4 \;=\; \log 128$$

Solution

$$x\log 2 + \log 4 \;=\; \log 128$$

$$\log 2^x + \log 4 \;=\; \log 128$$

$$\log(2^x \times 4) \;=\; \log 128$$

Take the antilog of both sides

$$2^x \times 4 = 128$$

$$2^x = \frac{128}{4}$$

$$2^x = 32$$

$$2^x = 2^5$$

Since the bases are the same equate the exponent

$$x \;=\; 5$$

EXAMPLE 15

Show clearly that

$$\log_a 36 + \frac{1}{2}\log_a 256 - 2\log_a 48 = -\log_a 4$$

Solution

$$\log_a 36 + \log_a 256^{\frac{1}{2}} - \log_a 48^2 = -\log_a 4$$

$$\log_a 36 + \log_a 16 - \log_a 2304 = -\log_a 4$$

$$\log_a 36 + \log_a 16 - \log_a 2304 = -\log_a 4$$

$$\log_a\left(\frac{36 \times 16}{2304}\right) \;=\; -\log_a 4$$

$$\log_a\left(\frac{576}{2304}\right) \;=\; -\log_a 4$$

$$\log_a \frac{1}{4} = -\log_a 4$$

$$\log_a 4^{-1} = -\log_a 4$$

$$-\log_a 4 = -\log_a 4$$

RHS = LHS

EXAMPLE 16

Solve the following Simultaneous equations

$$\log_{10} x + \log_{10} y = 1$$

$$\log_{10} x + 2\log_{10} y = 3$$

Solution

$$\log_{10} x + \log_{10} y = 1 -------- equ\ 1$$

$$\log_{10} x + \log_{10} y = \log_{10} 10$$

$$\log_{10} xy = \log_{10} 10$$

Taking the antilog of both sides

$$xy = 10 ----- eqn\ 1$$

$$\log_{10} x + 2\log_{10} y = 3 -------- eqn\ 2$$

$$\log_{10} x + 2\log_{10} y = \log_{10} 1000$$

$$\log_{10} x + \log_{10} y^2 = \log_{10} 1000$$

$$\log_{10} xy^2 = \log_{10} 1000$$

Taking the antilog of both sides

$$xy^2 = 1000 ----- eqn\ 2$$

Comparing equation 1 and 2

$$xy = 10 ----- eqn\ 1$$

$$xy^2 = 1000 ----- eqn\ 2$$

$$\frac{xy^2}{xy} = \frac{1000}{10}$$

y = 100

Substituting y into equation 1

$$x(100) = 10$$

$$x = \frac{1}{10}$$

Type equation here.

EXAMPLE 17

$\log_{10}(5x + 2) - \log_{10}(x - 1) = 0.7782$ find x

Solution

$$\log_{10}(5x + 2) - \log_{10}(x - 1) = 0.7782$$

$$\log_{10}\frac{(5x + 2)}{(x - 1)} = 0.7782$$

$$\frac{(5x + 2)}{(x - 1)} = 10^{0.7782}$$

$$\frac{(5x + 2)}{(x - 1)} = 6$$

$$5x + 2 = 6(x - 1)$$

$$5x + 2 = 6x - 6$$

$$5x - 6x = -6 - 2$$

-x = -8

x = 8

Example 18

Find the value of x for
$\log_3 x - 4\log_x 3 + 3 = 0$

Solution

$$\log_3 x - 4\log_x 3 + 3 = 0$$

Since the log are not in the same base, we change bases to make them the same

$$\log_x 3 = \frac{1}{\log_3 x}$$

$$\log_3 x - 4\left(\frac{1}{\log_3 x}\right) + 3 = 0$$

Let $\log_3 x = y$

$$y - 4\left(\frac{1}{y}\right) + 3 = 0$$

$$y - \left(\frac{4}{y}\right) + 3 = 0$$

$$y^2 - 4 + 3y = 0$$

$$y^2 + 3y - 4 = 0$$

$$y^2 + 4y - y - 4 = 0$$

$$y(y+4) - (y+4) = 0$$

$$(y-1)(y+4) = 0$$

y = 1 and y = -4

For y = 1

.. $\log_3 x = y$

. $\log_3 x = 1$

$x = 3$

For y = 1

.. $\log_3 x = y$

. $\log_3 x = -4$

$x = 3 - 4$

$$x = \frac{1}{3^4}$$

$$x = \frac{1}{81}$$