MASTERMIND SCHOLARS EDUCATIONAL ALLIANCE

GENERAL MATHEMATICS

CHANGE OF SUBJECT

LEARNING HOW TO MANIPULATE EQUATIONS

Change of subject in algebra means rewriting a formula so that a different variable becomes the subject, that is, expressing one variable in terms of the others.

In other words, we rearrange an equation so that the required variable is alone on one side of the equation.

Purpose of Changing the Subject

We change the subject of a formula in order to:

1. Make a particular variable easy to calculate

2. Substitute values easily

To change the subject of a formula, we use algebraic rules to undo operations applied to the variable.

Whatever you do to one side of the equation, you must do the same to the other side.

EXAMPLE 1

Make x the subject $ax + c = d + 2(d - x)$

Solution

Expand the bracket

$$ax + c = d + 2d - 2x$$

Group like terms of x

$$ax + 2x = d + 2d - c$$

Factorize x out

$$x(a + 2) = 3d - c$$

Divide both side

$$x = \frac{3d - c}{a + 2}$$

EXAMPLE 2

Make m the subject $mt + n = mp + q$

Solution

Group like terms of m

$$mt - mp = q - n$$

Factorize m out

$$m(t - p) = q - n$$

Divide both sides by $t - p$

$$m = \frac{q-n}{t-p}$$

EXAMPLE 3

Express l in terms of p, v and q in equation $3v - pq = 2(p + l)$

Solution

$$3v - pq = 2(p + l)$$

Expand the bracket

$$3v - pq = 2p + 2l$$

$$3v - pq - 2p = 2l$$

$$l = \frac{3v - pq - 2p}{2}$$

EXAMPLE 4

Express x in terms of the other variable in the equation $2px - ax = r + s$

Solution

$$2px - ax = r + s$$

$$x(2p - a) = r + s$$

$$x = \frac{r+s}{2p-a}$$

EXAMPLE 5

Make x the subject of the equation $3z = 5dx - 6$

Solution

$$3z = 5dx - 6$$

$$3z + 6 = 5dx$$

$$x = \frac{3z + 6}{5d}$$

EXAMPLE 6

Make x the subject of the formulae $3x(2 + a) - 4(5 - x) = r + 2x$

Solution

$$3x(2 + a) - 4(5 - x) = r + 2x$$

Expand the brackets

$$6x + 3ax - 20 + 4x = r + 2x$$

Group like terms of x

$$6x + 3ax + 4x - 2x = r + 20$$

$$3ax + 8x = r + 20$$

Factorize x out

$$x(3a + 8) = r + 20$$

$$x = \frac{r + 20}{3a + 8}$$

EXAMPLE 7

Make d the subject of the relation R = $\frac{h}{2} + \frac{d}{8h}$

Solution

Multiply through by the LCD

$$8h \times R = \frac{h}{2} \times 8h + \frac{d}{8h} \times 8h$$

$$8hR = 4h^2 + d$$

$$8hR - 4h^2 = d$$

$$4h(2R - h) = d$$

EXAMPLE 8

Make h the subject of the relation R = $\frac{h}{2} + \frac{d}{8h}$

Solution

Multiply through by the LCD

$$8h \times R = \frac{h}{2} \times 8h + \frac{d}{8h} \times 8h$$

$$8hR = 4h^2 + d$$

$$4h^2 - 8hR + d = 0$$

Since this is a quadratic , we simplify using the quadratic formular

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Standard quadratic formula $ax^2 + bx + c = 0$

By comparing with $4h^2 - 8hR + d = 0$

$a = 4 \qquad b = -8R \qquad c = d$

$$h = \frac{-(-8R) \pm \sqrt{(-8R)^2 - 4(4)(d)}}{2(4)}$$

$$h = \frac{8R \pm \sqrt{64R^2 - 16\mathrm{d}}}{8}$$

$$h = \frac{8R}{8} \pm \frac{\sqrt{64R^2 - 16\mathrm{d}}}{8}$$

$$h = R \pm \frac{\sqrt{64R^2 - 16\mathrm{d}}}{8}$$

$$h = R \pm \frac{\sqrt{16(4R^2 - \mathrm{d})}}{8}$$

EXAMPLE 9

Make x the subject of $\frac{x}{p} + q = c - \frac{x}{d}$

Solution

$$\frac{x}{p} + q = c - \frac{x}{d}$$

Multiply with the LCD

$$pd \times \frac{x}{p} + pd \times q = pd \times c - pd \times \frac{x}{d}$$

$$dx + pdq = pdc - px$$

Group like terms of x

$$dx + px = pdc - pdq$$

Factorize

$$x(d + p) = pd(c - q)$$

Divide by $d + p$

$$x = \frac{pd(c - q)}{(d + p)}$$

EXAMPLE 10

Make x the subject of $z = \frac{mx+c}{a+b}$

Solution

$$z = \frac{mx + c}{a + b}$$

$$z\,(a + b) = mx + c$$

$$z\,(a + b) - c = mx$$

$$x = \frac{z\,(a + b) - c}{m}$$

EXAMPLE 11

Make a the subject of $z = \frac{mx+c}{a+b}$

Solution

$$z = \frac{mx + c}{a + b}$$

$$z\,(a + b) = mx + c$$

$$az + bz = mx + c$$

$$az = mx + c - bz$$

$$a = \frac{mx + c - bz}{z}$$

EXAMPLE 12

Make u the subject of s = $\left(\frac{2u+at}{2}\right)t$

Solution

s = $\left(\frac{2u+at}{2}\right)t$

2s = $\frac{2u+at}{2} \times 2t$

2s = $(2u + at)t$

2s = 2ut + at^2

$2s - at^2 = 2ut$

u = $\frac{2s - at^2}{2t}$

EXAMPLE 13

Make v the subject of $t = 1 - \frac{3v}{vu-w}$

Solution

$$t = 1 - \frac{3v}{vu - w}$$

Multiply through by the LCD

t (vu - w) = (vu - w) $- \frac{3v}{vu-w} \times$ (vu - w)

$t\,(vu - w) = (vu - w) - 3v$

Expand brackets

$tvu - tw = vu - w - 3v$

You group like terms of v

$tvu - vu + 3v = tw - w$

Factor v out

$v\,(tu - u + 3) = w\,(t - 1)$

Divide through

$$v = \frac{w\,(t-1)}{u(t-1) + 3}$$

Example 14

Make u the subject of $d^2 + 3u = \frac{y}{x^2}$

Solution

$$d^2 + 3u = \frac{y}{x^2}$$

Multiply through with the LCD

$$x^2 \times d^2 + 3u \times x^2 = \frac{y}{x^2} \times x^2$$

$$x^2 d^2 + 3ux^2 = y$$

$$3ux^2 = y - x^2 d^2$$

$$u = \frac{y - x^2 d^2}{3x^2}$$

$$u = \frac{y - (dx)^2}{3x^2}$$

Example 15

Make y the subject of the relation :

$$\frac{m}{n-y} = \frac{n}{m+y}$$

Solution

$$\frac{m}{n-y} = \frac{n}{m+y}$$

$$m(m+y) = n(n-y)$$

$$m^2 + my = n^2 - ny$$

$$my + ny = n^2 - m^2$$

$$y\,(m + n) = n^2 - m^2$$

$$y = \frac{n^2 - m^2}{m + n}$$

$$y = \frac{(n+m)(n-m)}{m+n}$$

$$y = m - n$$

Example 16

Express p in terms of the other variables

$$q = \frac{y(x-p)}{m-p}$$

Solution

$$q = \frac{y(x-p)}{m-p}$$

$$q(m-p) = y(x-p)$$

$$qm - pq = xy - yp$$

$$yp - pq + = xy - qm$$

$$p(y-q) = xy - qm$$

$$p = \frac{xy-qm}{(y-q)}$$

Example 17

Given that $s = \frac{m^2(tp-n)}{2pn}$, make p the subject

Solution

$$s = \frac{m^2(tp-n)}{2pn}$$

$$2pns = m^2(tp-n)$$

$$2pns = m^2tp - m^2n$$

$$2pns - m^2tp = -m^2n$$

$$p(2ns - m^2t) = -m^2n$$

$$p = \frac{-m^2n}{2ns - m^2t}$$

Example 18

Express M in terms of T, W, P, S

$$T = WP\,[M^2 - (M - S)^2]$$

Solution

$$T = WP\,[M^2 - (M - S)^2]$$

$M^2 - (M - S)^2$ this is clearly an expression of difference of two squares

$$M^2 - (M - S)^2 = (M + M - S)(M - M + S)$$
$$M^2 - (M - S)^2 = (2M - S)S$$

$$T = WP(2M - S)S$$

$$T = WPS(2M - S)$$

$$T = 2WPSM - WPS^2$$

$$T + WPS^2 = 2WPSM$$

$$M = \frac{T + WPS^2}{2WPS}$$

Alternative solution

$$T = WP\,[M^2 - (M - S)^2]$$

$$T = WP\,[M^2 - (M - S)(M - S)]$$

$$T = WP\,[M^2 - (M^2 - 2MS + S^2)]$$

$$T = WP\,[M^2 - M^2 + 2MS - S^2]$$

$$T = WP\,[2MS - S^2]$$

$$T = 2WPMS - WPS^2$$

$$T + WPS^2 = 2WPMS$$

$$M = \frac{T + WPS^2}{2WPS}$$

EXAMPLE 19

Make y the subject of the relation $f = am + by^2$

Solution

$$f = am + by^2$$

$$f - am = by^2$$

$$\frac{f - am}{b} = y^2$$

$$y = \sqrt{\frac{f-am}{b}}$$

EXAMPLE 20

Make m the subject of the relation $y = \sqrt{\frac{f-am}{b}}$

Solution

$$y = \sqrt{\frac{f-am}{b}}$$

$$y^2 = \left(\sqrt{\frac{f - am}{b}}\right)^2$$

$$y^2 = \frac{f - am}{b}$$

$$by^2 = f - am$$

$$am = f - by^2$$

$$m = \frac{f - by^2}{a}$$

EXAMPLE 21

Make d the subject of the following relation

$$y = \sqrt{\frac{3dx}{8}}$$

solution

$$y^2 = \left(\sqrt{\frac{3dx}{8}}\right)^2$$

$$y^2 = \frac{3dx}{8}$$

$$8y^2 = 3dx$$

$$d = \frac{8y^2}{3x}$$

EXAMPLE 22

Express x in terms of t and π in the relation $t = 2\pi\sqrt{\frac{x}{g}}$

Solution

$$t^2 = \left(2\pi\sqrt{\frac{x}{g}}\right)^2$$

$$t^2 = 4\pi^2 \times \frac{x}{g}$$

$$t^2 = \frac{4\pi^2 x}{g}$$

$$t^2 g = 4\pi^2 x$$

$$x = \frac{t^2 g}{4\pi^2}$$

EXAMPLE 23

Make x the subject of d = $\sqrt{\frac{y}{x^2} - 3e}$

Solution

$$d = \sqrt{\frac{y}{x^2} - 3e}$$

$$d^2 = \left(\sqrt{\frac{y}{x^2} - 3e}\right)^2$$

$$d^2 = \frac{y}{x^2} - 3e$$

$$d^2 + 3e = \frac{y}{x^2}$$

$$x^2(d^2 + 3e) = y$$

$$x^2 = \frac{y}{d^2 + 3e}$$

$$x = \sqrt{\frac{y}{d^2 + 3e}}$$

EXAMPLE 24

Make x the subject of the relation

$$y = ab + \frac{c}{\frac{1}{x^2}}$$

Solution

$$y = ab + \frac{c}{\sqrt{x}}$$

$$y\sqrt{x} = ab\sqrt{x} + \frac{c}{\sqrt{x}} \times \sqrt{x}$$

$$y\sqrt{x} = ab\sqrt{x} + c$$

$$y\sqrt{x} - ab\sqrt{x} = c$$

$$\sqrt{x}\,(y - ab) = c$$

$$(\sqrt{x})^2 = \left(\frac{c}{y-ab}\right)^2$$

$$x = \left(\frac{c}{y-ab}\right)^2$$

EXAMPLE 25

Make z the subject of the relation

$$t = \sqrt{\frac{xy}{m} - z^2y}$$

Solution

$$t^2 = \left(\sqrt{\frac{xy}{m} - z^2y}\right)^2$$

$$t^2 = \frac{xy}{m} - z^2y$$

$$mt^2 = xy - z^2my$$

$$z^2my = xy - mt^2$$

$$z^2 = \frac{xy - mt^2}{my}$$

$$z = \sqrt{\frac{xy - mt^2}{my}}$$

EXAMPLE 26

Make u the subject of the relation

$$fx = w\left(\frac{v^2 - u^2}{2g}\right)$$

Solution

$$2gfx = w(v^2 - u^2)$$

$$2gfx = wv^2 - wu^2$$

$$wu^2 = wv^2 - 2gfx$$

$$u^2 = \frac{wv^2 - 2gfx}{w}$$

$$u = \sqrt{\frac{wv^2 - 2gfx}{w}}$$

EXAMPLE 27

Make y the subject of the relation $x = \sqrt[3]{\frac{y^2-p}{y^2+p}}$

Solution

$$x = \sqrt[3]{\frac{y^2 - p}{y^2 + p}}$$

$$x^3 = \left(\sqrt[3]{\frac{y^2 - p}{y^2 + p}}\right)^3$$

$$x^3 = \frac{y^2 - p}{y^2 + p}$$

$$x^3 (y^2 + p) = y^2 - p$$

$$x^3y^2 + x^3p = y^2 - p$$

$$x^3y^2 - y^2 = -p - x^3p$$

$$y^2(x^3 - 1) = -p - x^3p$$

$$y^2 = \frac{-p - x^3p}{x^3 - 1}$$

$$y^2 = \frac{p(-1 - x^3)}{x^3 - 1}$$

$$y = \sqrt{\frac{p(-1 - x^3)}{x^3 - 1}}$$

EXAMPLE 28

Make q the subject of the relation

$$r = \frac{f}{2} + \left(\frac{f^2}{4} + q^2\right)^{\frac{1}{2}}$$

Solution

Subtract $\frac{f}{2}$ from both sides

$$r - \frac{f}{2} = \left(\frac{f^2}{4} + q^2\right)^{\frac{1}{2}}$$

Square both side to remove bracket

$$\left(r - \frac{f}{2}\right)^2 = \left(\frac{f^2}{4} + q^2\right)^{\frac{1}{2}\times 2}$$

$$\left(r - \frac{f}{2}\right)^2 = \frac{f^2}{4} + q^2$$

Make q^2 stand alone

$$\left(r - \frac{f}{2}\right)^2 - \frac{f^2}{4} = q^2$$

$$q^2 = \left(r - \frac{f}{2}\right)^2 - \frac{f^2}{4}$$

Find the square root of both sides

$$q = \sqrt{\left(r - \frac{f}{2}\right)^2 - \frac{f^2}{4}}$$

EXAMPLE 29

Make y the subject of the relation

$$P = 2x\sqrt{\frac{q\left(1+\frac{r^2}{y^2}\right)}{s}}$$

SOLUTION

$$p^2 = \left(2x\sqrt{\frac{q\left(1+\frac{r^2}{y^2}\right)}{s}}\right)^2$$

$$p^2 = 4x^2\left[\frac{q\left(1+\frac{r^2}{y^2}\right)}{s}\right]$$

$$p^2 = 4x^2\frac{\left(\frac{qy^2+qr^2}{y^2}\right)}{s}$$

$$p^2 = 4x^2 \times \frac{qy^2+qr^2}{sy^2}$$

$$p^2 = \frac{4x^2(qy^2+qr^2)}{sy^2}$$

$$sy^2p^2 = 4x^2(qy^2 + qr^2)$$

$$sy^2p^2 = 4x^2y^2q + 4x^2r^2q$$

$$sy^2p^2 - 4x^2y^2q = 4x^2r^2q$$

$$y^2(sp^2 - 4x^2q) = 4x^2r^2q$$

$$y^2 = \frac{4x^2r^2q}{sp^2 - 4x^2q}$$

$$y = \sqrt{\frac{4x^2r^2q}{sp^2 - 4x^2q}}$$

EXAMPLE 30

Make h the subject of the relation

$$r = \frac{1}{3}\sqrt{\frac{6v - h^2}{\pi r}}$$

Solution

$$r^2 = \left(\frac{1}{3}\sqrt{\frac{6v - h^2}{\pi r}}\right)^2$$

$$r^2 = \frac{1}{9}\left(\frac{6v - h^2}{\pi r}\right)$$

Remove bracket

$$r^2 = \frac{6v - h^2}{9\pi r}$$

Multiply both sides by $9\pi r$

$$9\pi r \times r^2 = 6v - h^2$$
$$9\pi r^3 = 6v - h^2$$
$$9\pi r^3 - 6v = -h^2$$

Multiply through by -1

$$-9\pi r^3 + 6v = h^2$$

Square root both sides

$$\sqrt{6v - 9\pi r^3} = h$$

ASSIGNMENT

INSTRUCTIONS (READ CAREFULLY)

1. Write clearly at the top of your work: Name Course School

2. Solve all questions clearly on a white sheet of paper only.

3. Do not cancel, erase, or overwrite any part of your work. Any work with cancellations or heavy corrections may not be considered.

4. Show all working steps clearly.

5. Do NOT copy from classmates or any external source.

6. After completing your work: Take very clear pictures of each page and Scan your work using an app such as CamScanner.

7. Ensure: All pages are in the correct order and images are not blurred, cropped, or tilted

8. Submit your work through the link provided below.

DEADLINE : [MONDAY 11:59PM, 2nd February 2026],

1. For the equation, $k\,(x^2+4)+ky=\frac{7x^2+3}{2}$ make k the subject

2. Make μ the subject of the relation

$$E = v + \frac{1}{2}m\mu^2$$

3. Make p the subject of the relation

$$\frac{4^{x+1}}{x^3-x^2} = p(x^5-x^4)$$

4. If $y = \frac{2(\sqrt{x^2+m})}{3n}$ make x the subject of the formulae.

5. Make n the subject of the formula

QUESTION 31

Make n the subject of the relation

$$S = \frac{n}{2}[2a + (n-1)d]$$

SOLUTION

$$S = \frac{n}{2}[2a + (n-1)d]$$

$$2s = n[2a + (n-1)d]$$

$$2s = n[2a + nd - d]$$

$$2s = 2an + n^2d - nd$$

$$n^2d - nd + 2an - 2s = 0$$

$$n^2d - n(d - 2a) - 2s = 0$$

Comparing to the quadratic formulae

Since this is a quadratic , we simplify using the quadratic formular

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Standard quadratic formula $ax^2 + bx + c = 0$

By comparing with $n^2d - (d - 2a)n - 2s = 0$

$a = d \qquad b = -d + 2a \quad c = -2s$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-(-d + 2a) \pm \sqrt{(-d + 2a)^2 - 4(d)(-2s)}}{2(d)}$$

$$n = \frac{-(-d+2a) \pm \sqrt{(-d+2a)^2 + 8ds}}{2(d)}$$ final answer

Or choose to simplify more in the root

$$n = \frac{d - 2a \pm \sqrt{(-d + 2a)(-d + 2a) + 8ds}}{2\mathrm{d}}$$

$$n = \frac{d - 2a \pm \sqrt{d^2 - 2\mathrm{ad} - 2\mathrm{ad} + 4a^2 + 8ds}}{2\mathrm{d}}$$

$$n = \frac{d - 2a \pm \sqrt{d^2 - 4\mathrm{ad} + 4a^2 + 8ds}}{2\mathrm{d}}$$

$n = \frac{d-2a\pm\sqrt{4a^2+d(d-4a+8s)}}{2\mathrm{d}}$ final answer