MASTERMIND SCHOLARS EDUCATIONAL ALLIANCE

GENERAL MATHEMATICS

ALGEBRAIC EXPRESSION

Point 1 : Algebraic expressions are formed from variables and constants. We use the operations of addition, subtraction, multiplication and division on the variables and constants to form expressions.

For example, the expression $4xy + 7$ is formed from the variables x and y and constants 4 and 7.

The constant 4 and the variables x and y are multiplied to give the product $4xy$ and the constant 7 is added to this product to give the expression.

Point 2: Expressions are made up of terms. Terms are added to make an expression. For example, the addition of the terms $4xy$ and 7 gives the expression $4xy + 7$.

Point 3 : A term is a product of factors. The term $4xy$ in the expression $4xy + 7$ is a product of factors x, y and 4. Factors containing variables are said to be algebraic factors.

Point 4 : The coefficient is the numerical factor in the term. Sometimes anyone factor in a term is called the coefficient of the remaining part of the term.

Point 5 : Any expression with one or more terms is called a **polynomial.** Specifically a one term expression is called a **monomial;** a two-term expression is called a **binomial;** and a three-term expression is called a **trinomial.**

Point 6 : Terms which have the same algebraic factors are like terms. Terms which have different algebraic factors are unlike terms. Thus, terms 4xy and – 3xy are like terms; but terms 4xy and – 3x are not like terms.

EXAMPLE 1

$$(h^2 - k^2) - p(h + k)$$

Solution

$(h^2 - k^2)$ is a difference of two square

$$(h^2 - k^2) = (h + k)(h - k)$$

$$(h + k)(h - k) - p(h + k)$$

Now factorize

$$(h + k)(h - k - p)$$

EXAMPLE 2

Factorize completely the expression $(h + 2k)^2 + 4k^2 - h^2$

solution

$$(h + 2k)^2 + 4k^2 - h^2$$

$(4k^2 - h^2)$ is a difference of squares

$4k^2 - h^2 = (2k - h)(2k + h)$

$$(h + 2k)(h + 2k) + (2k - h)(2k + h)$$

Now factorize

$$(h + 2k)(h + 2k + 2k - h)$$

$$4k(h + 2k)$$

$$4kh + 8k^2$$

EXAMPLE 3

Factorize $3x^2 + 4x - 4$

Solution

$$3x^2 + 4x - 4$$

Multiply the co-efficient of the x^2 (first term) with the last term to obtain the number whose factors are to be used.

This give $3 \times 4 = -12$

Factors of 12 that adds to give 4

(12, 1)
(-2, 6)
(4, 3)

Add to get 4
multiply to get -12

(-2, 6)

$$3x^2 - 2x + 6x - 4$$

$$x(3x - 2) + 2(3x - 2)$$

$$(3x - 2)(x + 2)$$

EXAMPLE 4

Factorize $6x^2 + 5xy - 6y^2$

Solution

Multiply the co-efficient of the first term with the co-efficient of the last term

This give $6 \times 6 = 36$

Factors of 36 that adds to give 5

(1, 36)
(2, 18)
(3, 12)
(-4, 9)
(6, 6)

Add to get 5
multiply to get 36

(-4, 9)

$$6x^2 - 4xy + 9xy - 6y^2$$

$$2x(3x - 2y) + 3y(3x - 2y)$$

$$(3x - 2y)(2x + 3y)$$

EXAMPLE 5

Simplify

$$\boldsymbol{q^2 - (p - 2n)^2}$$

Solution

$$(q + p - 2n)[q - (p - 2n)]$$

$$(q + p - 2n)[q - p + 2n]$$

EXAMPLE 6

Simplify

$$(\boldsymbol{2m+3k})^{\boldsymbol{2}}-(\boldsymbol{m-2k})^{\boldsymbol{2}}$$

Solution

$$(2m+3k+\ m-2k)[(\ 2m+3k-\ (m-2k)]$$

$$(3m+k)[(\ 2m+3k-\ m+2k)]$$

$$(3m+k)(\ m+5k)$$

EXAMPLE 7

Simplify

$$\boldsymbol{\pi R^2 h-\pi r^2 h}$$

Solution

$$\pi h\ (R^2-r^2)$$

$$\pi h\ (R-r)(R+r)$$

EXAMPLE 8

$$5a^2 - 16ab + 3b^2$$

Solution

Multiply the co-efficient of the first term with the co-efficient of the last term.

$$5 \times 3 = 15$$

Factors of 15

(-1, -15)
(3, 5)

Multiply to get 15

Add to get -16

$$5a^2 - ab - 15ab + 3b^2$$

$$a\,(5a - b) - 3b\,(5a - b)$$

$$(5a - b)(a - 3b)$$

EXAMPLE 9

$$(x + 3y)^2 + 6xy - x^2$$

Solution

$$(x + 3y)^2 + 6xy - x^2$$

$$(x + 3y)\,(x + 3y) + 6xy - x^2$$

$$x^2 + 3xy + 3xy + 9y^2 + 6xy - x^2$$

$$6xy + 9y^2 + 6xy$$

$$9y^2 + 12xy$$

$$3y\,(3y + 4x)$$

EXAMPLE 10

$$(m+n)^2 - m^2 + 2n^2$$

Solution

$$(m+n)(m+n) - m^2 + 2n^2$$

$$m^2 + mn + mn + n^2 - m^2 + 2n^2$$

$$m^2 + 2mn + 3n^2 - m^2$$

$$2mn + 3n^2$$

$$3n^2 + 2mn$$

$$n\,(3n + 2m)$$

EXAMPLE 11

$$3a^2b^2 + 13ab - 30$$

Solution

Multiply the co-efficient of the first term with the co-efficient of the last term

-90

(1, 90)

(2, 45)

(3, 30)

(-5, 18)

(6, 15)

(9, 10)

$$3a^2b^2 + 13ab - 30$$

$$3a^2b^2 - 5ab + 18ab - 30$$

$$ab\,(3ab - 5) + 6\,(3ab - 5)$$

(3ab - 5)(ab + 6)

EXAMPLE 12

By how much is the sum of $3x$, $(6x - 5), 9x$ and $(4x + 1)$ less than $30x$?

Solution

$$3x + (6x - 5) + 9x + (4x + 1)$$

$$3x + 6x - 5 + 9x + 4x + 1$$

$$3x + 6x + 9x + 4x + 1 - 5$$

$$22x - 4$$

$22x - 4$ is less than $30x$

We need to subtract $22x - 4$ from $30x$

$$30x - (22x - 4)$$

$$30x - 22x + 4$$

$$8x + 4$$

EXAMPLE 13

By how much is $(a + 2b - c)$ exceed $(2b - a - c)$?

Solution

To determine how much larger one expression or number is than another, you must always subtract the smaller value from the larger one.
This difference tells you by how much the greater quantity exceeds the lesser.

$$(a + 2b - c) - (2b - a - c)$$

$$(a + 2b - c - 2b + a + c)$$

$$2a$$

EXAMPLE 14

What is the coefficient of , x^2 and x^3 in the expansion of $(4x^2 + 3x - 1)(3x + 1)$

Solution

$$(4x^2 + 3x - 1)(3x + 1)$$

$$12x^3 + 4x^2 + 9x^2 + 3x - 3x - 1$$

$$12x^3 + 13x^2 - 1$$

The co-efficient of x^2 and x^3 is given as 12 and 13 respectively.

EXAMPLE 15

Expand $(3x - y)(3x + y) - (3x + 2y)(3x - 2y)$

Solution

$$(9x^2 + 3xy - 3xy - y^2) - (9x^2 - 6xy + 6xy - 4y^2)$$

$$(9x^2 + 3xy - 3xy - y^2 - 9x^2 + 6xy - 6xy + 4y^2)$$

$$4y^2 - y^2$$

$$3y^2$$

Alternatively

$$(3x - y)(3x + y) - (3x + 2y)(3x - 2y)$$

$$[(3x)^2 - y^2] - [(3x)^2 - (2y)^2]$$

$$[9x^2 - y^2] - [9x^2 - 4y^2]$$

$$9x^2 - y^2 - 9x^2 + 4y^2$$

$$-y^2 + 4y^2$$

$$4y^2 - y^2$$

$$3y^2$$

EXAMPLE 16

Expand $(2x + 2y)(x - y) + (2x - 2y)(x + y)$

Solution

$$(2x + 2y)(x - y) + (2x - 2y)(x + y)$$

$$(2x^2 - 2xy + 2xy - 2y^2) + (2x^2 + 2xy - 2xy - 2y^2)$$

$$2x^2 - 2xy + 2xy - 2y^2 + 2x^2 + 2xy - 2xy - 2y^2$$

$$2x^2 - 2y^2 + 2x^2 - 2y^2$$

$$4x^2 - 4y^2$$

$$4(x^2 - y^2)$$

$$4(x - y)(x + y)$$

Alternatively

$$(2x + 2y)(x - y) + (2x - 2y)(x + y)$$

$$2(x + y)(x - y) + 2(x - y)(x + y)$$

$$2(x^2 - y^2) + 2(x^2 - y^2)$$

$$2x^2 - 2y^2 + 2x^2 - 2y^2$$

$$2x^2 + 2x^2 - 2y^2 - 2y^2$$

$$4x^2 - 4y^2$$

$$4(x^2 - y^2)$$

$$4(x-y)(x+y)$$

EXAMPLE 17

Simplify $\frac{2x-1}{3}-\frac{x+3}{2}$

Solution

$$\frac{2x-1}{3}-\frac{x+3}{2}$$

Find your LCM

$$\frac{2(2x-1)-3(x+3)}{6}$$

$$\frac{4x-2-3x-9}{6}$$

$$\frac{x-11}{6}$$

EXAMPLE 18

Simplify $\frac{3}{x+1}-\frac{2}{x-1}-\frac{5x}{x^2-1}$

Solution

$$\frac{3}{x+1}-\frac{2}{x-1}-\frac{5x}{x^2-1}$$

$$\frac{3(x-1)-2(x+1)-5x}{(x+1)(x-1)}$$

$$\frac{3x-3-2x-2-5x}{(x+1)(x-1)}$$

$$\frac{x-5-5x}{(x+1)(x-1)}$$

$$\frac{-4x-5}{(x+1)(x-1)}$$

$$\frac{-4x-5}{x^2-1}$$

EXAMPLE 19

Simplify $\frac{3m}{9m^2-1}-\frac{1}{2(3m+1)}$

Solution

$$\frac{3m}{(3m+1)(3m-1)}-\frac{1}{2(3m+1)}$$

$$\frac{6m-(3m-1)}{2(3m+1)(3m-1)}$$

$$\frac{6m-(3m-1)}{2(3m+1)(3m-1)}$$

$$\frac{6m-3m+1}{2(3m+1)(3m-1)}$$

$$\frac{3m+1}{2(3m+1)(3m-1)}$$

$$\frac{3m+1}{2(3m-1)}$$

EXAMPLE 20

$$\frac{m+1}{m-1}-\frac{m-1}{m+1}+\frac{4}{m^2-1}$$

Solution

$$\frac{(m+1)(m+1)-(m-1)(m-1)+4}{m^2-1}$$

$$\frac{m^2+2m+1-(m^2-2m+1)+4}{m^2-1}$$

$$\frac{m^2+2m+1-m^2+2m-1+4}{m^2-1}$$

$$\frac{4m+4}{m^2-1}$$

$$\frac{4(m+1)}{(m+1)(m-1)}$$

$$\frac{4}{m-1}$$

EXAMPLE 21

Simplify the Expression

$$\frac{\frac{6x+5}{x-5}-1}{\frac{3x+20}{x-5}+2}$$

Solution

$$\left(\frac{6x+5}{x-5}-1\right)\div\left(\frac{3x+20}{x-5}+2\right)$$

$$\left(\frac{6x+5}{x-5}-\frac{1}{1}\right)\div\left(\frac{3x+20}{x-5}+\frac{2}{1}\right)$$

$$\frac{6x+5-(x-5)}{x-5} \div \frac{3x+20+2(x-5)}{x-5}$$

$$\frac{6x+5-x+5}{x-5} \div \frac{3x+20+2x-10}{x-5}$$

$$\frac{5x+10}{x-5} \div \frac{5x+10}{x-5}$$

Reciprocate

$$\frac{5x+10}{x-5} \times \frac{x-5}{5x+10}$$

$$1$$

EXAMPLE 22

$$Simplify\ \frac{3x^2-3y^2}{12x} \div \frac{x+y}{4x}$$

Solution

$$\frac{3x^2-3y^2}{12x} \div \frac{x+y}{4x}$$

$$\frac{3(x^2-y^2)}{12x} \div \frac{x+y}{4x}$$

$$\frac{3(x-y)(x+y)}{12x} \div \frac{x+y}{4x}$$

Reciprocate

$$\frac{3(x-y)(x+y)}{12x} \times \frac{4x}{x+y}$$

$$\frac{3(x-y)}{3}$$

$$x-y$$

Example 23

Simplify $\frac{a^2-4b^2}{a^2+4ab+4b^2}$

Solution

$$\frac{a^2-4b^2}{a^2+4ab+4b^2}$$

$$\frac{(a-2b)(a+2b)}{a^2+4ab+4b^2}$$

$$\frac{(a-2b)(a+2b)}{(a+2b)(a+2b)}$$

$$\frac{(a-2b)}{(a+2b)}$$

Example 24

Simplify $\frac{a^2b^4-a^4b^2}{ab(a+b)}$

Solution

$$\frac{a^2b^4 - a^4b^2}{ab(a+b)}$$

$$\frac{(ab)^2(b^2 - a^2)}{ab(a+b)}$$

$$\frac{(ab)^2(b+a)(b-a)}{ab(a+b)}$$

$$\frac{(ab)^2(a+b)(b-a)}{ab(a+b)}$$

$$ab(b-a)$$

$$ab^2 - a^2b$$

Example 25

$$m^2 - 2mn + n^2 - 9r$$

Solution

$$m^2 - 2mn + n^2 - 9r$$

$$m^2 - mn - mn + n^2 - 9r$$

$$m(m-n) - n(m-n) - 9r$$

$$(m-n)(m-n) - 9r$$

$$(m-n)^2 - 9r$$

$$(m-n)^2 - (3\sqrt{r})^2$$

$$(m-n+3\sqrt{r})\,(m-n-3\sqrt{r})$$

EXAMPLE 26

1. Two tanks X and Y are filled to capacity with petrol. Tank X holds 600 litres more than tank Y. If 100 litres of petrol were pumped out of each tank, tank X would then contain 3 times as much petrol as tank Y. Find the capacity of each tank.

Solution

Let x be the amount of petrol in the tanks

Tank X = $(x + 600) - 100$

Tank Y = $x - 100$

$$(x + 600) - 100 = 3(x - 100)$$

$$x + 600 - 100 = 3x - 300$$

$$x + 500 = 3x - 300$$

$$500 + 300 = 3x - x$$

$$800 = 2x$$

$$x = 400$$

Tank x = $x + 600$

Tank y = x

Tank x = $400 + 600 = 1000$

Tank y = 400

EXAMPLE 27

2. Ama is m years old now and Fatau is y years older than Ama. If (x -5) years ago, Fatou was twice as old as Ama, express their ages in terms of y and m.

Solution

Ama = $m - (x - 5)$

Fatau = $m + y - (x - 5)$

$$2[m - (x - 5)] = m + y - (x - 5)$$

$$2(m - x + 5) = m + y - x + 5$$

$$2m - 2x + 10 = m + y - x + 5$$

$$2m - m - 2x + x - y = 5 - 10$$

$$m - x + -y = -5$$

$$-x = -5 - m + y$$

$$x = m - y + 5$$

EXAMPLE 28

The present age of a woman is 30 years older than her daughter. 15 years ago, she was twice as old as her daughter. How old is her daughter at present ?

Solution

Woman = (x + 30) - 15

Daughter = x - 15

$$(x + 30) - 15 = 2(x - 15)$$

$$x + 30 - 15 = 2x - 30$$

$$x + 15 = 2x - 30$$

$$15 + 30 = 2x - x$$

$$45 = x$$

$$x = 45$$

Woman = x + 30

45 + 30

75

Daughter = x

45

EXAMPLE 29

The total age of 2 sisters is 108. One is 18 years older than the other. Find their ages.

Sister A = x + 18

Sisiter B = x

$$x + 18 + x = 108$$

$$2x + 18 = 108$$

$$2x = 108 - 18$$

$$2x = 90$$

$$x = 45$$

Sister A = x + 18

45 + 18

63

Sister B = 45

Example 30

A mother is 3 times as old as the daughter, 9 years ago the mother was 4 times as old as her daughter. Find their ages.

Solution

Mother = 3x - 9

Daughter = x - 9

$$3x - 9 = 4(x - 9)$$

$$3x - 9 = 4x - 36$$

$$-9 + 36 = 4x - 3x$$

x = 27

Mother = 3x

3(27)

81

Daughter = x

27

Example 31

Kofi and Kweku are two brothers. Kofi is older than Kweku. Given that Kofi's age is (5x – 4) years and kweku's age is (2x +1)years.

a. Write down an expression in terms of x for how much older, kofi is than kweku.

b. If kofi is 10 years older than kweku, calculate

i. the value of x

ii. The ages of kofi and kweku

Solution

Kofi is older that kweku

Kofi = (5x – 4)

Kweku = (2x +1)

$$(5x – 4) – (2x + 1)$$

$$5x – 4 -2x -1$$

$$5x -2x - 4 -1$$

$$3x - 5$$

i. $3x – 5 = 10$

$3x = 10 + 5$

$3x = 15$

$x = 5$

or

$(5x - 4) = (2x + 1) + 10$

$5x – 4 = 2x + 1 + 10$

$5x – 4 = 2x + 11$

$5x – 2x = 11 + 4$

$3x = 15$

$x = 5$

Kofi = $(5x – 4)$

$5(5) – 4$

$25 – 4$

21

Kweku = $(2x + 1)$

$2(5) + 1$

$10 + 1$

11

Example 32.

The sum of the ages of Akeem and Dope is 35years. The sum of twice Akeem's age and 3 times Dope's age is 89. Find their present age.

Solution

Akeem = x

Dope = y

x + y = 35

Akeem = 2x

Dope = 3y

2x + 3y = 89

x+y = 35 ---------- eqn 1

2x + 3y = 89 ---------- eqn 2

Using substitution method

Make x the subject from eqn 1

x = 35 –y

Substitute into equation 2

2x + 3y = 89

2(35 - y) + 3y = 89

70 – 2y + 3y = 89

70 + y = 89

y = 89 - 70

y = 19

Subtitute y into the easiest equation

x +y = 35

x + 19 = 35

x = 35 – 19

x = 16

Akeem's age = 16 years

Dope's age = 19 years

Example 33

The sum of the ages of sarpong and kotey is 50 years. In 5 years time, the age of kotey will be 4 times the age of sarpong. How old is each boy.

Solution

Sarpong = x

Kotey = y

x + y = 50

Sarpong = x + 5

Kotey = y + 5

4(x + 5) = y + 5

$$x + y = 50 \text{ ------ equation 1}$$

$$4(x + 5) = y + 5$$

$$4x + 20 = y + 5$$

$$4x - y = 5 - 20$$

$$4x - y = -15 \text{---------eqn 2}$$

$$x + y = 50$$

$$4x - y = -15$$

Using Elimination method

$$5x = 35$$

$$x = 7$$

Subtitute your answer into the easiest equation

$$x + y = 50$$

$$7 + y = 50$$

$$y = 50 - 7$$

$$y = 43$$

Sarpong = 7

Kotey = 43

Example 34

Agyapong is three times as old as Atsu, three years ago, he was four times as old as Atsu. How old is each boy now ?

Try this

Example 35

Mr Jibril is four times as old as his son. Four years ago, he was seven times as old as his son. In how many years will Mr Jibril's age be twice his son's age.

Try this

Example 36

Ama's age is two – third that of her elder sister. Four years ago, Ama's age was half the sister's. How old is the sister ?

Try this

Example 36

Adu takes 10 days less than the number of days that Mr Kubi takes to build an article. If both of them take 12 days to build the article together, how long will it take each of them to build the article alone.

SOLUTION

Let the number of days x

Mr. Kubi takes = x days.
Then Adu takes: $x - 10$ days M1

$$\text{Work rate} = \frac{1}{number\ of\ days}$$

$$\text{Rate of Mr Kubi} = \frac{1}{x}$$

$$\text{Rate of Adu} = \frac{1}{x-10} \quad \text{M1}$$

$$\frac{1}{x} + \frac{1}{x-10} = \frac{1}{12} \quad \text{M1}$$

$$\frac{x - 10 + x}{x(x-10)} = \frac{1}{12}$$

$$\frac{2x-10}{x(x-10)} = \frac{1}{12}$$

$$12(2x-10) = x(x-10)$$

$$24x - 120 = x^2 - 10x$$

$$x^2 - 34x - 120 = 0 \qquad \text{M1}$$

$$(x-30)(x-4)$$

$x - 30 = 0$
$x = 30$

$x - 4 = 0$
$x = 4$

If $x = 4$, then Adu would take: $\mathbf{4 - 10 = -6}$ days(not possible) , So we reject $x = 4$.

Mr Kubi = 30 days

Adu = 30 – 10 A1

20

IGNORE THIS PLEASE

MASTERMIND SCHOLARS EDUCATIONAL ALLIANCE
ASSIGNMENT 01
ALGEBRAIC EXPRESSION

INSTRUCTIONS (READ CAREFULLY)

1. **Write clearly at the top of your work:**
   Name
   Course
   School

2. **Solve all questions clearly** on a **white sheet of paper only**.
3. **Do not cancel, erase, or overwrite** any part of your work. Any work with cancellations or heavy corrections may not be **considered.**
4. **Show all working steps clearly**.
5. **Do NOT copy** from classmates or any external source.
6. After completing your work: Take **very clear pictures** of each page **and Scan your work** using an app such as **CamScanner**.
7. Ensure: All pages are in the **correct order** and images are **not blurred**, cropped, or tilted
8. **Submit your work through the link provided below**.

DEADLINE : [FRIDAY 11:59PM, MIDNIGHT], DEADLINE : [FRIDAY 11:59PM, MIDNIGHT]

QUESTIONS

1. Factorize $3(a-b)^2+14(a-b)-5$ **[3 MARKS]**

2. Factorize $4(2a-3)^2-9(a-1)^2$ **[3 MARKS]**

3. **By how much does the expression** $5a-2b+3c$ **exceed one-third the sum of** $3a+6b-c$ and $6a-3b+4c$?

[4 MARKS]