MASTERMIND SCHOLARS EDUCATIONAL ALLIANCE

RIGID MOTION (TRANSFORMATION)

Transformation involves the process of changes in shape, size and position of plane figures in the Cartesian plane.

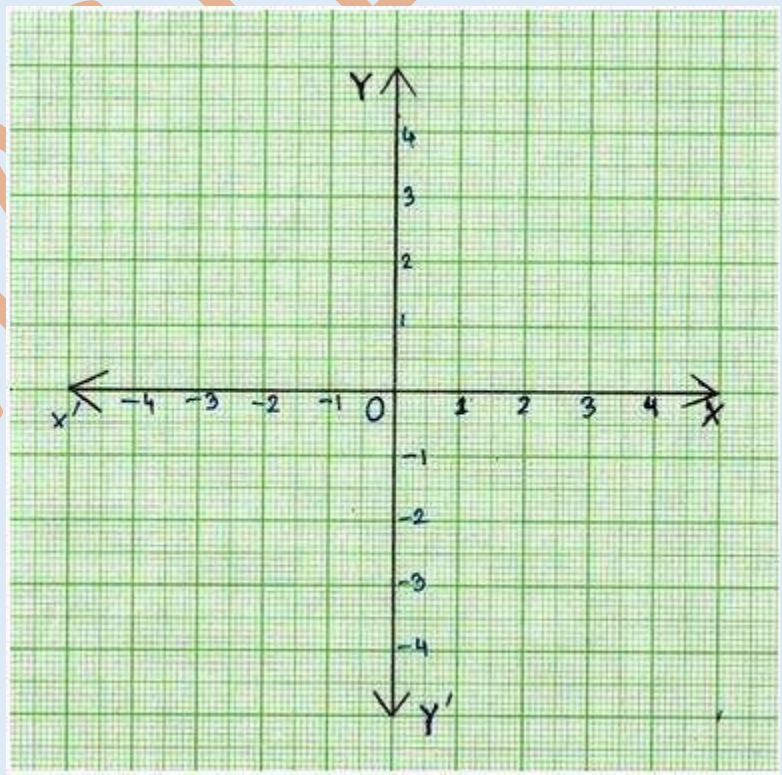
Rigid motion is involved in only changes in position. The processes which can bring about transformation include

1. Reflection
2. Rotation
3. Translating by a vector
4. Enlargement and reduction
5. Special transformation

Reflection

Reflection in the Cartesian plane can be done in 6 different ways.

1. Reflection in the x – axis (reflection in the line y = 0)



Rule for reflection in the x – axis (y = 0)

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ -y \end{pmatrix}$$

Find the image A' of the point A(5, 2) under the reflection in the line y = 0 (x - axis)

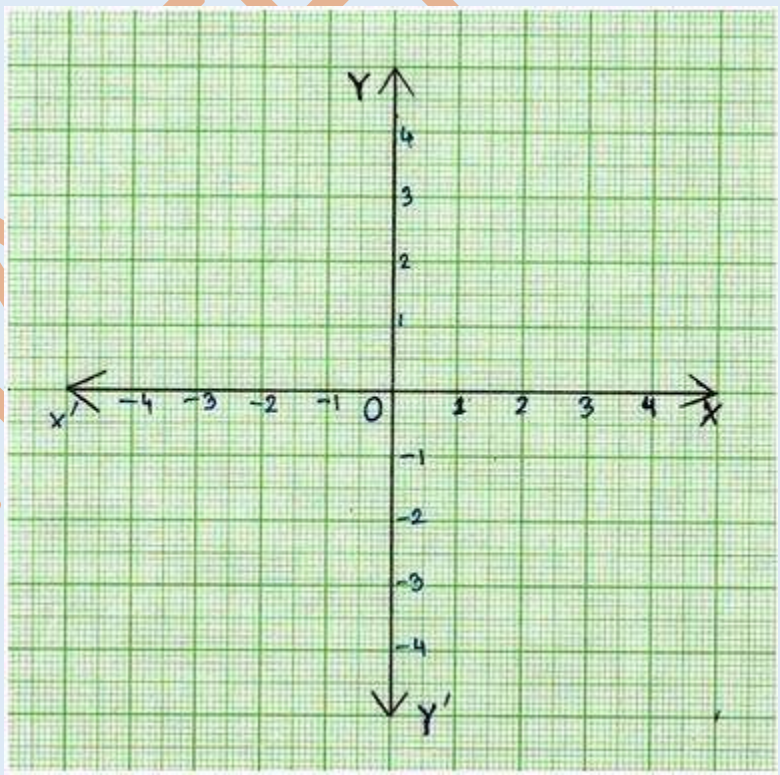
Solution

For reflection in the x – axis ,

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ -y \end{pmatrix}$$

$$A\begin{pmatrix} 5 \\ 2 \end{pmatrix} \rightarrow A'\begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

2. Reflection in the y – axis (reflection in the line x = 0)



Rule for reflection in the y– axis (x = 0)

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ y \end{pmatrix}$$

Find the image A' of the point A(5, 2) under the reflection in the line x = 0 (y - axis)

Solution

For reflection in the x – axis ,

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ y \end{pmatrix}$$

$$A\begin{pmatrix} 5 \\ 2 \end{pmatrix} \rightarrow A'\begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

3. Reflection in the line y = k (y – k = 0) where k is a constant

The line y = k behaves like the x – axis, so if the point (x , y) is reflected in the line y = k , the the image is by

For reflection in the line y = k,

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ 2k - y \end{pmatrix}$$

Example

Find the image of the point A(3,5) under the reflection in the line y – 2 = 0 .

Solution

y – 2 = 0

y = 2

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ 2k - y \end{pmatrix}$$

$$A\begin{pmatrix} 3 \\ 5 \end{pmatrix} \rightarrow A'\begin{pmatrix} 3 \\ 2(2) - 5 \end{pmatrix}$$

$$A\begin{pmatrix} 3 \\ 5 \end{pmatrix} \rightarrow A'\begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

4. Reflection in the line x = k (x – k = 0) where k is a constant

The line x = k behaves like the y – axis, so if the point (x , y) is reflected in the line x = k , then the image is by

For reflection in the line y = k,

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2k - x \\ y \end{pmatrix}$$

Example

Find the image of the point A(3,5) under the reflection in the line y – 2 = 0 .

Solution

y – 2 = 0

y = 2

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ 2k - y \end{pmatrix}$$

$$A\begin{pmatrix} 3 \\ 5 \end{pmatrix} \rightarrow A'\begin{pmatrix} 3 \\ 2(2) - 5 \end{pmatrix}$$

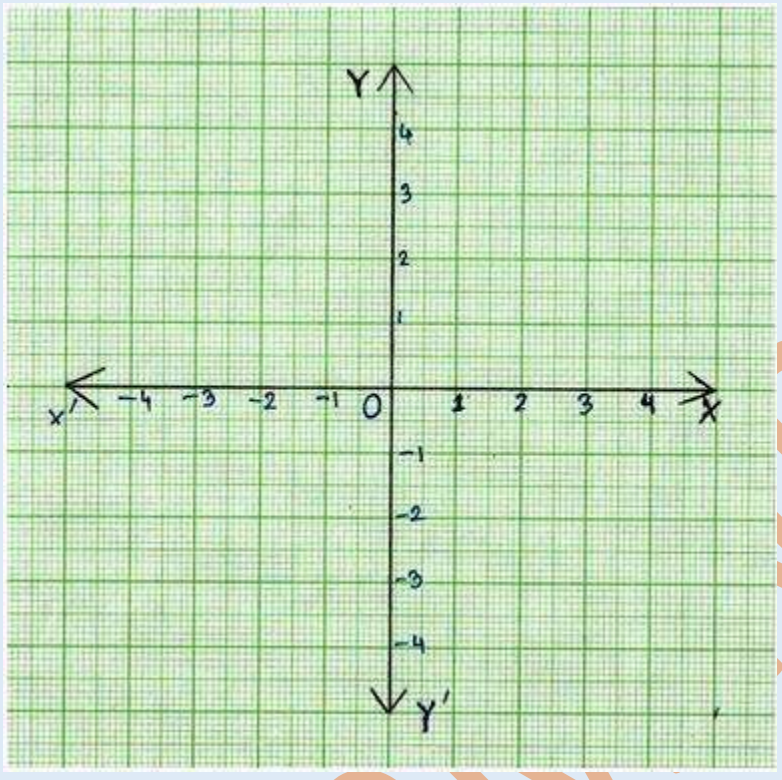
$$A\begin{pmatrix} 3 \\ 5 \end{pmatrix} \rightarrow A'\begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

5. Reflection in the line y = x (y – x = 0)

The line y = x is a diagonal line with gradient (slope) equal to 1 and passes though the origin (0,0).

If a point is reflected in the line y =x , then the image is given by

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ x \end{pmatrix}$$



Example 1

Find the image of the point Q(-2 , 6) under the reflection in the line y – x = 0

Solution

Y – x = o

y = x

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ x \end{pmatrix}$$

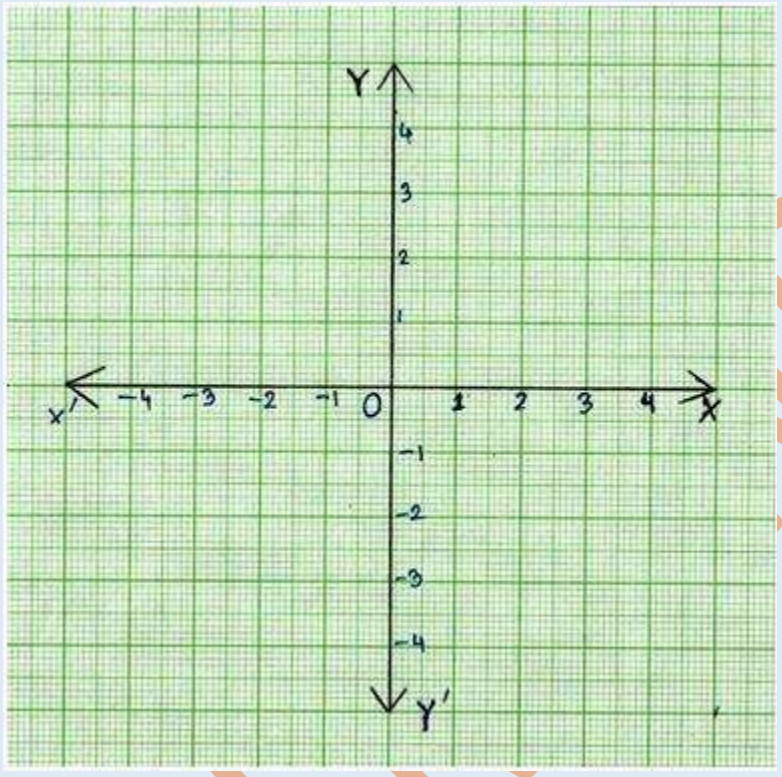
$$Q\begin{pmatrix} -2 \\ 6 \end{pmatrix} \rightarrow Q'\begin{pmatrix} 6 \\ -2 \end{pmatrix}$$

6. Reflection in the line y = - x , (y + x = 0)

The line y = -x is a diagonal line with gradient (slope) equal to -1 and passes though the origin (0,0).

If a point is reflected in the line y = -x , then the image is given by

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ -x \end{pmatrix}$$



Example 1

Find the image of the point Q(-2 , 6) under the reflection in the line y – x = 0

Solution

y – x = o

y = x

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ -x \end{pmatrix}$$

$$Q\begin{pmatrix} -2 \\ 6 \end{pmatrix} \rightarrow Q'\begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

Example 1

a. Using a scale of 2cm to 2units on each axis, draw on a sheet of graph paper two perpendicular axes OX and OY for the intervals $-12 \leq x \leq 12$ and $-12 \leq y \leq 12$.

b. Draw ΔABC with coordinates A(5,7), B(3,4) and C(7,3). Draw the image $\Delta A_1B_1C_1$ of ΔABC under a reflection in the x-axis, where A → A_1 , B → B_1 , C → C_1.

C. Draw the image $\Delta A_2B_2C_2$ of ΔABC under a reflection in the line x + 2 = 0 where A → A_2 , B → B_2 , C → C_2.

d. Draw the image $\Delta A_3B_3C_3$ of ΔABC under a reflection in the line y = -x where A → A_3 , B → B_3 , C → C_3.

Solution

Thing to look out for when plotting your points on the graph

1. Make sure you write down your scale on the topmost corner of the graph

2. Make sure you label your coordinate anytime you plot

b.

For reflection in the x axis

$(x, y) \rightarrow (x, -y)$

A(5,7) → $A_1(5, -7)$

B(3,4) → $B_1(3, -4)$

C(7,3) → $C_1(7, -3)$

c.

For reflection in the line x + 2 = 0

x = -2

$(x, y) \rightarrow (2k - x,\ y)$

A(5,7) → $(2(-2) - 5, 7)$

A_2 (-9 , 7)

B(3,4) → $(2(-2) - 3, 4)$

B_2 (-7 , 4)

C(7,3) → $(2(-2) - 7, 3)$

C_2 (-11, 3)

ROTATION

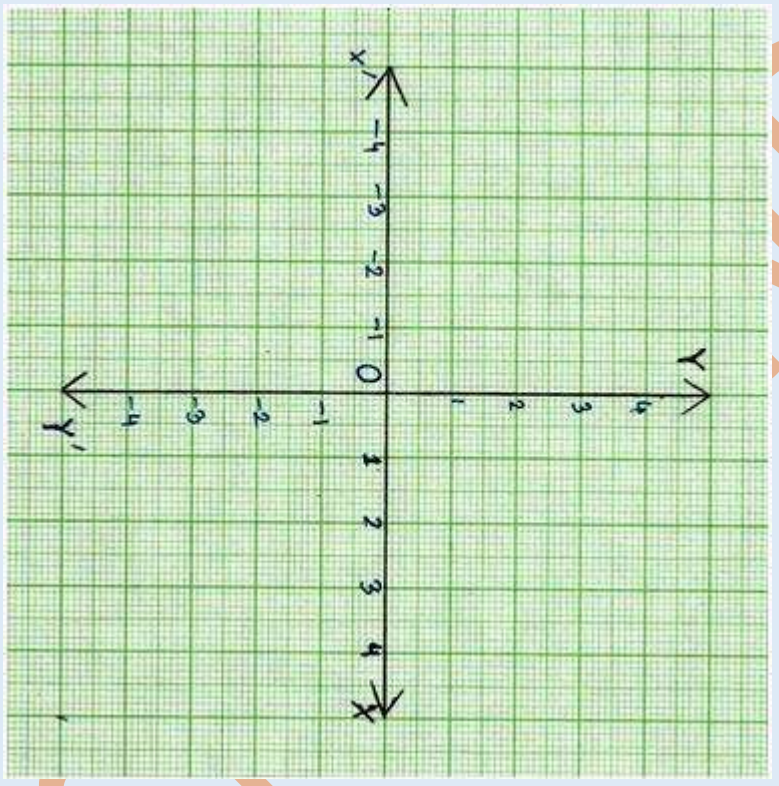
Rotation takes place about an axis which can be the origin or any point other than the origin.

The direction for rotation is either clockwise (+) or anticlockwise (-)

The angle of rotation are $\pm 90°, \pm 180°, \pm 270°$

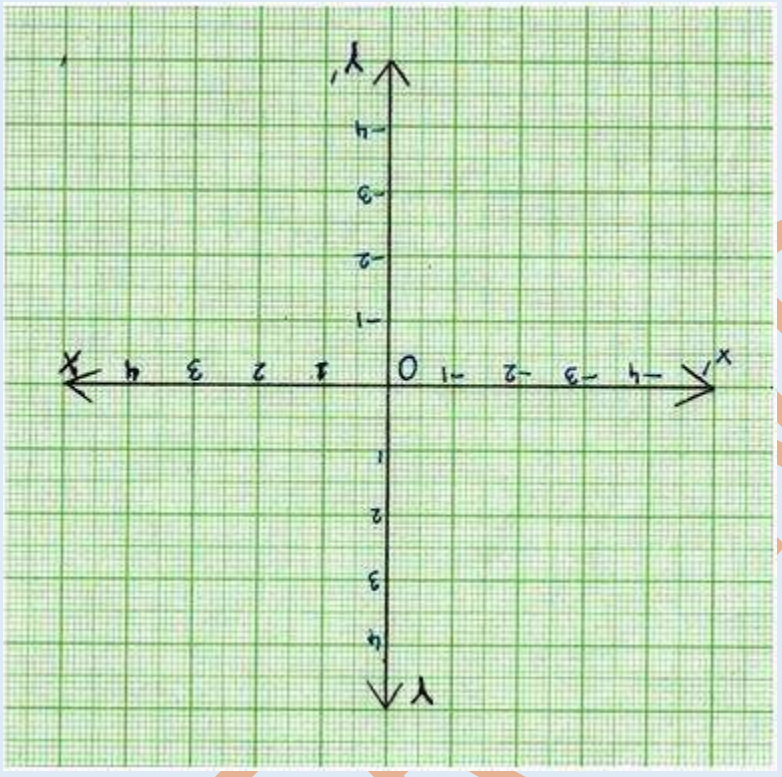
Clockwise

Rotating in the clockwise 90 about the origin.



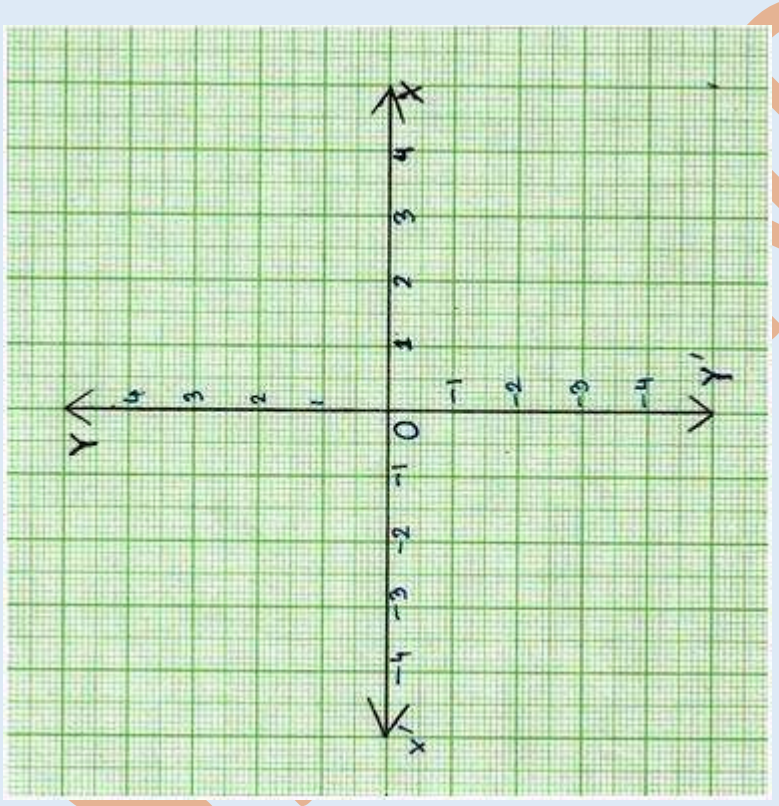
$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ -x \end{pmatrix}$$

Rotating in the clockwise 180 about the origin.



$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \end{pmatrix}$$

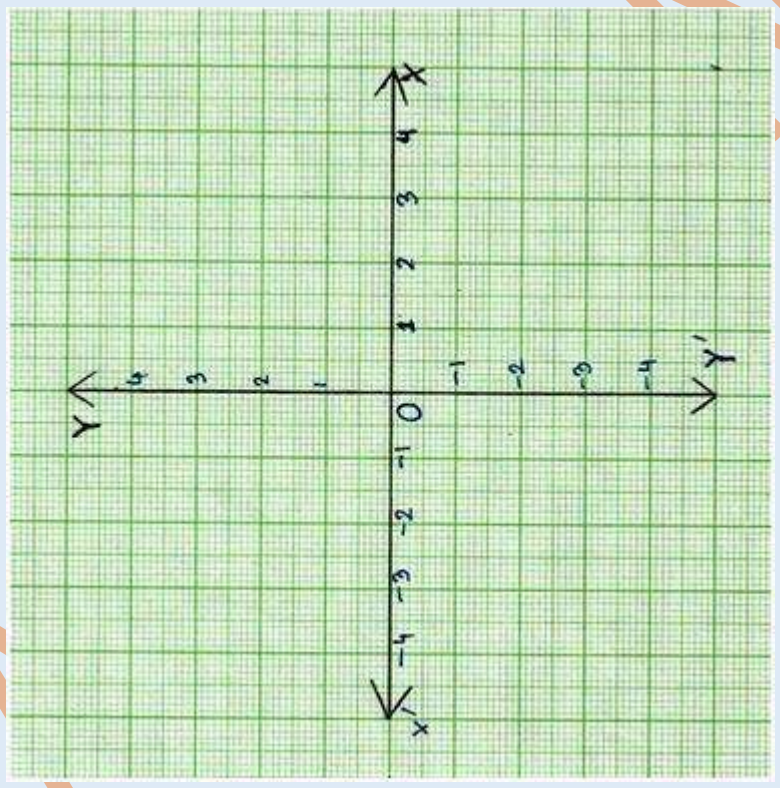
Rotating in the clockwise 270 about the origin.



$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$$

Anticlockwise

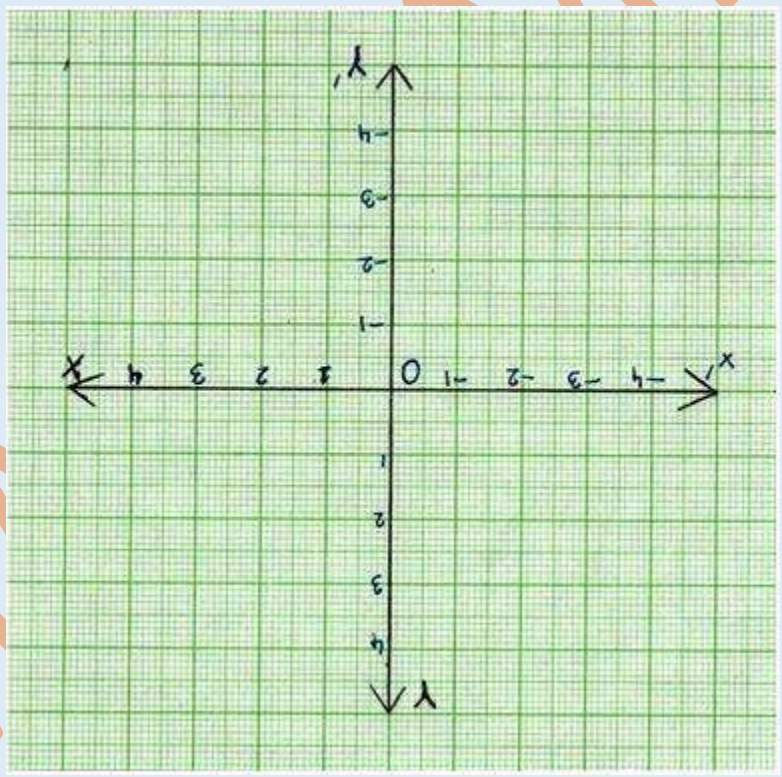
Rotating in the anticlockwise 90 about the origin.



$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$$

Note that clockwise 270 and anticlockwise 90 are the same

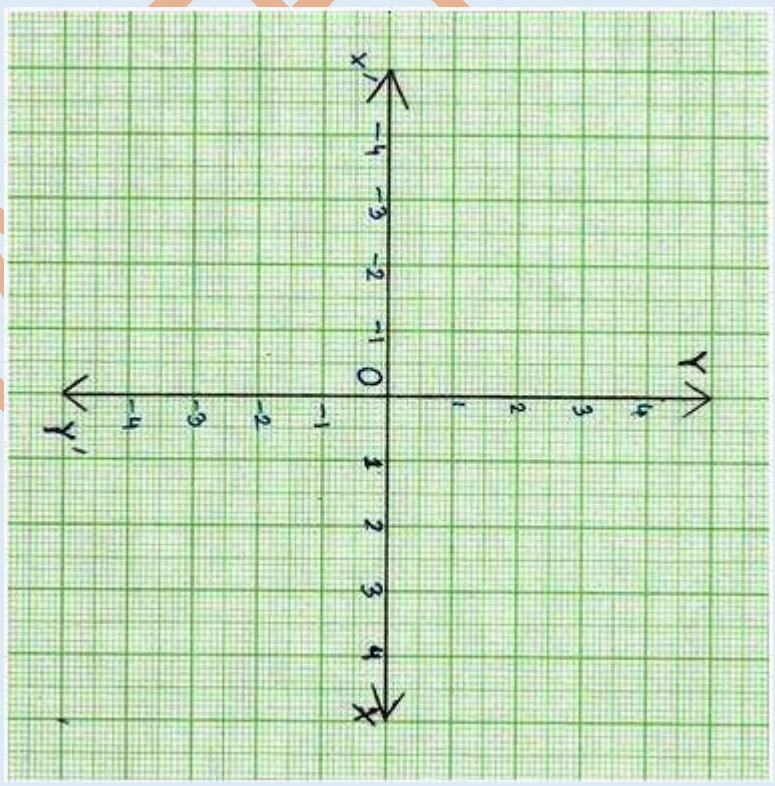
Rotating in the anticlockwise 180 about the origin.



$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \end{pmatrix}$$

Note that clockwise 180 and anticlockwise 180 are the same

Rotating in the anticlockwise 270 about the origin.



Anticlockwise 270 and clockwise 90 are the same.

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ -x \end{pmatrix}$$

IN SUMMARY

Clockwise (90°) or Anticlockwise (270°) about the origin

1. $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ -y \end{pmatrix}$

Antilockwise (90°) or clockwise (270°) about the origin

2. $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -x \\ y \end{pmatrix}$

Antilockwise (180°) or clockwise (180°) about the origin

3. $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -x \\ -y \end{pmatrix}$

Example

a. Using a scale of **2 cm to 2 units** on both axes, draw two perpendicular axes **OX** and **OY** for the intervals $-10 \leq x \leq 10$ and $-12 \leq y \leq 12$.

b. Draw

i. Quadrilateral **ABCD** with coordinates A(2,6), B(4,1), C(6,3), D(5,6)

ii. The image quadrilateral **A′B′C′D′** of ABCD under a **clockwise rotation of 90° about the origin**, where A → A′, B → B, C → C′ , D → D′

iii. The image quadrilateral **A″B″C″D″** of ABCD under an **anticlockwise rotation of 90° about the origin**, where A → A″, B → B″, C → C″ , D → D″

iv. The image quadrilateral **A‴ B‴ C‴ D‴** of ABCD under a half turn rotation **about the origin**, where A → A‴, B → B‴, C → C‴, D → D‴

Rotation about a point other than the origin.

If the point (x , y) is rotated about the point (m, n) then an image can be found by following the steps below

i. Subtract the centre of rotation (m, n) from the point to be rotated (x, y)

$$\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} x-m \\ y-n \end{pmatrix}$$

ii. Apply the appropriate rotation (i.e clockwise 90^o and 270^o, anticlockwise 90^o and 270^o and half – turn 180^0)

$$\begin{pmatrix} x-m \\ y-n \end{pmatrix} \longrightarrow \begin{pmatrix} y-n \\ -(x-m) \end{pmatrix}$$ [90 clockwise]

iii. Add the result in step (ii) above the centre of rotation (m, n)

$$\begin{pmatrix} y-n \\ -(x-m) \end{pmatrix} + \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} y-n+m \\ -(x-m)+n \end{pmatrix}$$

You can remember easily using the acronym **SAA** ie. Subtract , Add, Add

Example

Find the image of the point A(5,7) through a clockwise rotation of 90^0 about the point (2,8)

Solution

i. Subtract the centre of rotation (2, 8) from the point to be rotated (5, 7)

$$\begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 8 \end{pmatrix} = \begin{pmatrix} 5-2 \\ 7-8 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

ii. Apply the appropriate rotation (i.e clockwise 90^o)

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$ [90 clockwise]

iii. Add the result in step (ii) above the centre of rotation (2, 8)

$$\begin{pmatrix} -1 \\ -3 \end{pmatrix} + \begin{pmatrix} 2 \\ 8 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

TRANSLATION

Translation by a vector $\begin{pmatrix} m \\ n \end{pmatrix}$

If the point (a,b) is translated by the vector $\begin{pmatrix} m \\ n \end{pmatrix}$, then the image is given by $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} a+m \\ b+n \end{pmatrix}$

Point + Vector = image

Therefore

i. Point = Image – Vector
ii. Vector = Image – point

Example

Find the image of the point (5,3) under a translating by a vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Solution

$$\begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$$

Example 2

The image of the point A(7,2) under a translation by the vector V is A′**(-2,3). Find the vector V**

Solution

$$\begin{pmatrix} 7 \\ 2 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -9 \\ 5 \end{pmatrix}$$

ENLARGEMENT AND REDUCTION

If the point (a,b) undergo an enlargement or a reduction from the origin with scale factor (k), then the image is given by

$$\begin{pmatrix} a \\ b \end{pmatrix} \longrightarrow \begin{pmatrix} k \times a \\ k \times b \end{pmatrix}$$

$$\begin{pmatrix} ka \\ kb \end{pmatrix}$$

Where k is a constant

i. If $|k| > 1$ the transformation is an enlargement.

ii. If $0 < |k| < 1$ the transformation is a reduction.

Example

Find the image of the point A(2,4) under the enlargement with scale factor k = $\frac{3}{2}$ about the origin.

Special Transformation

A special transformation involves an expression in the form of a mapping which does not obey any of the rules of transformation.

Example